RAILROAD CURVES

AND

EARTHWORK

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SIXTH EDITION, REVISED

McGRAW-HILL BOOK COMPANY, Inc.
NEW YORK: 239 WEST 39TH STREET
LONDON: 6 & 8 BOUVERIE ST., E. C. 4
1920

Norwood Press
J. S. Cushing Co. — Berwick & Smith Co.
Norwood, Mass., U.S.A.

PREFACE.

THIS book was prepared for the use of the students in the author's classes. It has been used in lithographed sheets for a number of years in very nearly the present form, and has given satisfaction sufficient to suggest putting it in print. An effort has been made to have the demonstrations simple and direct, and special care has been given to the arrangement and the typography, in order to secure clearness and conciseness of mathematical statement. Much of the material in the earlier part of the book is necessarily similar to that found in one or more of several excellent field books, although the methods of demonstration are in many cases new. This will be found true especially in Compound Curves, for which simple treatment has been found quite possible. New material will be found in the chapters on Turnouts and on "Y" Tracks and Crossings. The Spiral Easement Curve is treated originally. The chapters on Earthwork are essentially new; they include Staking Out; Computation, directly and with Tables and Diagrams; also Haul, treated ordinarily and by Mass Diagram. Most of the material relating to Earthwork is not elsewhere readily available for students' use.

The book has been written especially to meet the needs of students in engineering colleges, but it is probable that it will be found useful by many engineers in practice. The size of page allows it to be used as a pocket book in the field. It is difficult to avoid typographical and clerical errors; the author will consider it a favor if he is notified of any errors found to exist.

C. FRANK ALLEN.

BOSTON,
September, 1899.

PREFACE TO FIFTH EDITION

The revision of this edition has been extensive. Few pages dealing with curves have escaped some change. In considerable part it has been a matter of refining or clearing up points shown by teaching to admit of improvement. A considerable amount of new material has been added and a few less important problems omitted; by rearrangement, and condensation in places, the size of the book has not been appreciably increased. The chapter on Turnouts has been almost completely rewritten; railroad practice in Turnouts has progressed materially in late years and complete revision of this chapter seemed advisable. The chapter on Connecting Tracks and Crossings has been materially extended. The chapter on Spirals has largely been rewritten and adapted to the use of the Spiral of the American Railway Engineering Association, the merits of which appeal to the author aside from the official sanction which establishes it as standard. A few, but not many, important changes have been made in the chapters on Earthwork.

It is still true that while this text was prepared primarily for students, nevertheless this book has proved to be well adapted to the requirements of the practicing railroad engineer or other engineer who has to deal with curves or with earthwork computation.

January, 1914. C. FRANK ALLEN.

PREFACE TO SIXTH EDITION

Highway practice follows, in many ways, railroad practice in laying out curves and computing earthwork, but there are some features of difference; and the subject of Circular Arcs, which received original treatment in the last edition, has been carried further.

In the computation of earthwork, some methods new to text-books have been added; these have come to the author from the practice in "valuation work."

A few pages have been added on "haul." Many pages have had perfecting changes, and simpler treatment of some subjects has been found worth while.

April, 1920. C. FRANK ALLEN.

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RAILROAD CURVES AND EARTHWORK.

CHAPTER I.

1. The operations of "locating" a railroad, as commonly practiced in this country, are three in number : —

I. Reconnoissance.
II. Preliminary Survey.
III. Location Survey.

I. RECONNOISSANCE.

2. The Reconnoissance is a rapid survey, or rather a critical examination of country, without the use of the ordinary instruments of surveying. Certain instruments, however, are used, the Aneroid Barometer, for instance. It is very commonly the case that the termini of the railroad are fixed, and often intermediate points also. It is desirable that no unnecessary restrictions as to intermediate points should be imposed on the engineer to prevent his selecting what he finds to be the best line, and for this reason it is advisable that the reconnoissance should, where possible, precede the drawing of the charter.

3. The first step in reconnoissance should be to procure the best available maps of the country ; a study of these will generally furnish to the engineer a guide as to the routes or section of country that should be examined. If maps of the United States Geological Survey are at hand, with contour lines and other topography carefully shown, the reconnoissance can be largely determined upon these maps. Lines clearly impracticable will be thrown out, the maximum grade closely determined, and the field examinations reduced to a minimum No

route should be accepted finally from any such map, but a careful field examination should be made over the routes indicated on the contour maps. The examination, in general, should cover the general section of country, rather than be confined to a single line between the termini. A straight line and a straight grade from one terminus to the other is desirable, but this is seldom possible, and is in general far from possible. If a single line only is examined, and this is found to be nearly straight throughout, and with satisfactory grades, it may be thought unnecessary to carry the examination further. It will frequently, however, be found advantageous to deviate considerably from a straight line in order to secure satisfactory grades. In many cases it will be necessary to wind about more or less through the country in order to secure the best line. Where a high hill or a mountain lies directly between the points, it may be expected that a line around the hill, and somewhat remote from a direct line, will prove more favorable than any other. Unless a reasonably direct line is found, the examination, to be satisfactory, should embrace all the section of intervening country, and all feasible lines should be examined.

4. There are two features of topography that are likely to prove of especial interest in reconnoissance, *ridge lines* and *valley lines.*

A *ridge line* along the whole of its course is higher than the ground immediately adjacent to it on each side. That is, the ground slopes downward from it to both sides. It is also called a *watershed line.*

A *valley line*, to the contrary, is lower than the ground immediately adjacent to it on each side. The ground slopes upward from it to both sides. Valley lines may be called *water-course lines.*

A *pass* is a place on a ridge line lower than any neighboring points on the same ridge. Very important points to be determined in reconnoissance are the passes where the ridge lines are to be crossed; also the points where the valleys are to be crossed; and careful attention should be given to these points. In crossing a valley through which a large stream flows, it may be of great importance to find a good bridge crossing. In some cases where there are serious difficulties in crossing a ridge, a tunnel may be necessary. Where such structures, either

bridges or tunnels, are to be built, favorable points for their construction should be selected and the rest of the line be compelled to conform. In many parts of the United States at the present time, the necessity for avoiding grade crossings causes the crossings of roads and streets to become governing points of as great importance as ridges and valleys.

5. There are several purposes of reconnoissance: first, to find whether there is any satisfactory line between the proposed termini; second, to establish which of several lines is best; third, to determine approximately the maximum grade necessary to be used; fourth, to report upon the character or geological formation of the country, and the probable cost of construction depending somewhat upon that; fifth, to make note of the existing resources of the country, its manufactures, mines, agricultural or natural products, and the capabilities for improvement and development of the country resulting from the introduction of the railroad. The report upon reconnoissance should include information upon all these points. It is for the determination of the third point mentioned, the rate of maximum grade, that the barometer is used. Observing the elevations of governing points, and knowing the distances between those points, it is possible to form a good judgment as to what rate of maximum grade to assume.

6. The Elevations are usually taken by the *Aneroid Barometer.* Tables for converting barometer readings into elevations above sea-level are readily available and in convenient form for field use. (See Table XI., Allen's Field and Office Tables.)

Distances may be determined with sufficient accuracy in many cases from the map, where a good one exists. Where this method is impossible or seems undesirable, the distance may be determined in one of several different ways. When the trip is made by wagon, it is customary to use an *Odometer*, an instrument which measures and records the number of revolutions of the wheel to which it is attached, and thus the distance traveled by the wagon. There are different forms of odometer. In its most common form, it depends upon a hanging weight or pendulum, which is supposed to hold its position, hanging vertical, while the wheel turns. The instrument is attached to the wheel between the spokes and as near to the hub as practicable. At low speeds it registers accurately; as the

speed is increased, a point is reached where the centrifugal force neutralizes or overcomes the force of gravity upon the pendulum, and the instrument fails to register accurately, or perhaps at high speeds to register at all. If this form of odometer is used, a clear understanding should be had of the conditions under which it fails to correctly register. A theoretical discussion might closely establish the point at which the centrifugal force will balance the force of gravity. The wheel striking against stones in a rough road will create disturbances in the action of the pendulum, so that the odometer will fail to register accurately at speeds less than that determined upon the above assumption.

A cyclometer, manufactured for automobile use, is connected both with the wheel and the axle, and so measures positively the relative motion between the wheel and axle, and this ought to be reliable for registering accurately. Many engineers prefer to count the revolutions of the wheel themselves, tying a rag to the wheel to make a conspicuous mark for counting.

When the trip is made on foot, pacing will give satisfactory results. An instrument called the *Pedometer* registers the results of pacing. As ordinarily constructed, the graduations read to quarter miles, and it is possible to estimate to one-tenth that distance. Pedometers are also made which register paces. In principle, the pedometer depends upon the fact that, with each step, a certain shock or jar is produced as the heel strikes the ground, and each shock causes the instrument to register. Those registering miles are adjustable to the length of pace of the wearer.

If the trip is made on horseback, it is found possible to get good results with a steady-gaited horse, by first determining his rate of travel and figuring distance by the time consumed in traveling. Excellent results are said to have been secured in this way.

7. It is customary for engineers not to use a compass in reconnoissance. although this is sometimes done in order to trace the line traversed upon the map, and with greater accuracy. A pocket level will be found useful. The skillful use of pocket instruments will almost certainly be found of great value to the engineer of reconnoissance.

It may, in cases, occur that no maps of any value are in existence or procurable. It may be necessary, in such a case, to make a rapid instrumental survey, the measurements being taken either by pacing, chain, or stadia measurements. This is, however, unusual.

8. The preliminary survey is based upon the results of the reconnoissance, and the location upon the results of the preliminary survey. The reconnoissance thus forms the foundation upon which the location is made. Any failure to find a suitable line and the best line constitutes a defect which no amount of faithfulness in the later work will rectify. The most serious errors of location are liable to be due to imperfect reconnoissance; an inefficient engineer of reconnoissance should be avoided at all hazards. In the case of a new railroad, it would, in general, be proper that the Chief Engineer should in person conduct this survey. In the case of the extension of existing lines, this might be impracticable or inadvisable, but an assistant of known responsibility, ability, and experience should in this case be selected to attend to the work.

CHAPTER II.

II. PRELIMINARY SURVEY.

9. The Preliminary Survey is based upon the results of the reconnoissance. It is a survey made with the ordinary instruments of surveying. Its purpose is to fix and mark upon the ground a first trial line approximating as closely to the proper final line as the difficulty of the country and the experience of the engineer will allow; further than this, to collect data such that this survey shall serve as a basis upon which the final Location may intelligently be made. In order to approximate closely in the trial line, it is essential that the maximum grade should be determined or estimated as correctly as possible, and the line fixed with due regard thereto.

It will be of value to devote some attention here to an explanation about Grades and "Maximum Grades."

10. Grades. — The ideal line in railroad location is a straight and level line. This is seldom, if ever, realized. When the two termini are at different elevations, a line straight and of uniform grade becomes the ideal. It is commonly impossible to secure a line of uniform grade between termini. In operating a railroad, an engine division will be about 100 miles, sometimes less, often more. In locating any 100 miles of railroad, it is almost certain that a uniform grade cannot be maintained. More commonly there will be a succession of hills, part of the line up grade, part down grade. Sometimes there will be a continuous up grade, but not at a uniform rate. With a uniform grade, a locomotive engine will be constantly exerting its maximum pull or doing its maximum work in hauling the longest train it is capable of hauling; there will be no power wasted in hauling a light train over low or level grades upon which a heavier train could be hauled. Where the grades are not uniform, but are rising or falling, or rising irregularly, it will be found that the topography on some particular 5 or 10

miles is of such a character that the grade here must be steeper than is really necessary anywhere else on the line; or there may be two or three stretches of grade where about the same rate of grade is necessary, steeper than elsewhere required. The steep grade thus found necessary at some special point or points on the line of railroad is called the "Maximum Grade" or "Ruling Grade" or "Limiting Grade," it being the grade that limits the weight of train that an engine can haul over the whole division. It should then be the effort to make the rate of maximum grade as low as possible, because the lower the rate of the maximum grade, the heavier the train a given locomotive can haul, and because it costs not very much more to haul a heavy train than a light one. The maximum grade determined by the reconnoissance should be used as the basis for the preliminary survey. How will this affect the line? Whenever a hill is encountered, if the maximum grade be steep, it may be possible to carry the line straight, and over the hill; if the maximum grade be low, it may be necessary to deflect the line and carry it around the hill. When the maximum grade has been once properly determined, if any saving can be accomplished by using it rather than a grade less steep, the maximum grade should be used. It is possible that the train loads will not be uniform throughout the division. It will be advantageous to spend a small sum of money to keep any grade lower than the maximum, in view of the *possibility* that at this particular point the train load will be heavier than elsewhere on the division. Any saving made will in general be of one or more of three kinds:—

a. Amount or "quantity" of excavation or embankment;
b. Distance;
c. Curvature.

11. In some cases, a satisfactory grade, a low grade for a maximum, can be maintained throughout a division of 100 miles in length, with the exception of 2 or 3 miles at one point only. So great is the value of a low maximum grade that all kinds of expedients will be sought for, to pass the difficulty without increasing the rate of maximum grade, which we know will apply to the whole division.

12. Sometimes by increasing the length of line, we are able to reach a given elevation with a lower rate of grade. Some-

times heavy and expensive cuts and fills may serve the purpose. Sometimes all such devices fail, and there still remains necessary an increase of grade at this one point, but at this point only. In such case it is now customary to adopt the higher rate of grade for these 2 or 3 miles and operate them by using an extra or additional engine. In this case, the "ruling grade" for the division of 100 miles is properly the "maximum grade" prevailing over the division generally, the higher grade for a few miles only being known as an "Auxiliary Grade" or more commonly a "Pusher Grade." The train which is hauled over the engine division is helped over the auxiliary or pusher grade by the use of an additional engine called a "Pusher." Where the use of a short "Pusher Grade" will allow the use of a low "maximum grade," there is evident economy in its use. The critical discussion of the importance or value of saving distance, curvature, rise and fall, and maximum grade, is not within the scope of this book, and the reader is referred to Wellington's "Economic Theory of Railway Location."

13. The Preliminary Survey follows the general line marked out by the reconnoissance, but this rapid examination of country may not have fully determined which of two or more lines is the best, the advantages may be so nearly balanced. In this case two or more preliminary surveys must be made for comparison. When the reconnoissance has fully determined the general route, certain details are still left for the preliminary survey to determine. It may be necessary to run two lines, one on each side of a small stream, and possibly a line crossing it several times. The reconnoissance would often fail to settle minor points like this. It is desirable that the preliminary survey should closely approximate to the final line, but it is not important that it should fully coincide anywhere.

An important purpose of the "preliminary" is to provide a map which shall show enough of the topography of the country, so that the Location proper may be projected upon this map. Working from the line of survey as a base line, measurements should be taken sufficient to show streams and various natural objects as well as the contours of the surface.

14. The Preliminary Survey serves several purposes: —

First. To fix accurately the maximum grade for use in Location.

Second. To determine which of several lines is best.

Third. To provide a map as a basis upon which the Location can properly be made.

Fourth. To make a close estimate of the cost of the work.

Fifth. To secure, in certain cases, legal rights by filing plans.

15. It should be understood that the preliminary survey is, in general, simply a means to an end, and rapidity and economy are desirable. It is an instrumental survey. Measurements of distance are taken usually with the chain, although a tape is sometimes used. Angles are taken generally with a transit; some advocate the use of a compass. The line is ordinarily run as a broken line with angles, but is occasionally run with curves connecting the straight stretches, generally for the reason that a map of such a line is available for filing, and certain legal rights result from such a filing. With a compass, no backsight need be taken, and, in passing small obstacles, a compass will save time on this account. A transit line can be carried past an obstacle readily by a zigzag line. Common practice among engineers favors the use of the transit rather than the compass. Stakes are set, at every "Station," 100 feet apart, and the stakes are marked on the face, the first 0, the next 1, then 2, and so to the end of the line. A stake set 1025 feet from the beginning would be marked 10 + 25.

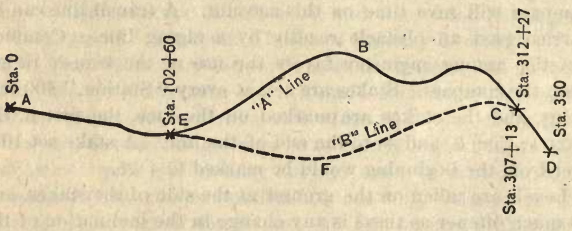
Levels are taken on the ground at the side of the stakes, and as much oftener as there is any change in the inclination of the ground. All the surface heights are platted on a profile, and the grade line adjusted.

16. The line should be run from a governing point towards country allowing a choice of location, that is from a pass or from an important bridge crossing, towards country offering no great difficulties. There is an advantage in running from a summit downhill, subject, however, to the above considerations. In running from a summit down at a prescribed rate of grade, an experienced engineer will carry the line so that, at the end of a day's work, the levels will show the line to be about where it ought to be. For this purpose, the levels must be worked up and the profile platted to date at the close of each day. Any slight change of line found necessary can then be made early the next morning. A method sometimes adopted in working down from a summit is for the locating engineer to

plat his grade line on the profile, daily in advance, and then during the day, plat a point on his profile whenever he can conveniently get one from his leveler, and thus find whether his line is too high or too low.

17. Occasionally the result of two or three days' work will yield a line extremely unsatisfactory, enough so that the work of these two or three days will be abandoned. The party "backs up" and takes a fresh start from some convenient point. In such case the custom is not to tear out several pages of note-book, but instead to simply draw a line across the page and mark the page "Abandoned." At some future time the abandoned notes may convey useful information to the effect that this line was attempted and found unavailable. In general, all notes worth taking are worth saving.

Sometimes after a line has been run through a section of country, there is later found a shorter or better line.



In the figure used for illustration, the first line, "A" Line, is represented by AEBCD, upon which the stations are marked continuously from A to D, 350 stations. The new line, "B" Line, starts from E, Sta. 102 + 60, and the stationing is held continuous from O to where it connects with the "A" Line at C. The point C is Sta. 312 + 27 of the "A" Line, and is also Sta. 307 + 13 of the "B" Line. It is not customary to restake the line from C to D in accordance with "B" Line stationing. Instead of this, a note is made in the note-books as follows: —

Sta. 312 + 27 "A" Line = 307 + 13 "B" Line.

Some engineers make the note in the following form: —

Sta. 307 to 313 = 86 ft.

The first form is preferable, being more direct and less liable to cause confusion.

18. All notes should be kept clearly and nicely in a note-book — never on small pieces of paper. The date and the names of members of the party should be entered each day in the upper left-hand corner of the page. An office copy should be made as soon as opportunity offers, both for safety and convenience. *The original notes should always be preserved;* they would be admissible as evidence in a court of law where a copy would be rejected. When two or more separate or alternate lines are run, they may be designated

Line "A," Line "B," Line "C,"

or "A" Line, "B" Line, "C" Line.

19. The Organization of Party may be as follows: —

1. Locating Engineer.	
2. Transitman.	Transit Party.
3. Head Chainman.	
4. Stakeman.	
5. Rear Chainman.	
6. Back Flag.	
7. Axemen (one or more).	
8. Leveler.	Level Party.
9. Rodman (sometimes two).	
10. Topographer.	Topographical Party.
11. Assistant.	
12. Cook.	
13. Teamster.	

20. The Locating Engineer is the chief of party, and is responsible for the business management of the camp and party, as well as for the conduct of the survey. He determines where the line shall run, keeping ahead of the transit, and establishing points as foresights or turning-points for the transitman. In open country, the extra axeman can assist by holding the flag at turning-points, and thus allowing the locating engineer to push on and pick out other points in advance. The locating engineer keeps a special note-book or memorandum book; in it he notes on the ground the quality of material, rock, earth, or whatever it may be; takes notes to determine the lengths and positions of bridges, culverts, and other structures; shows the localities of timber, building stones, borrow pits, and

other materials valuable for the execution of the work; in fact, makes notes of all matters not properly attended to by the transit, leveling, or topography party. The rapid and faithful prosecution of the work depend upon the locating engineer, and the party ought to derive inspiration from the energy and vigor of their chief, who should be the leader in the work. In open and easy country, the locating engineer may instill life into the party by himself taking the place of the head chainman occasionally. In country of some difficulty, his time will be far better employed in prospecting for the best line.

21. The Transitman does the transit work, ranges in the line from the instrument, measures the angles, and keeps the notes of the transit survey. The following is a good form for the left-hand page of the note-book: —

Station	Point	Deflection	Observed Bearing	Calculated Bearing
7			N 3° 30′ E	N 3° 38′ E
6	⊙ + 24	33° 02′ R		
5			N 29° 30′ W	N 29° 24′ W
4	⊙	12° 09′ L		
3				
2				
1			N 17° 15′ W	N 17° 15′ W
0	⊙			

Notes of topography and remarks are entered on the right-hand page, which, for convenience, is divided into small squares by blue lines, with a red line running up and down through the middle.

The stations run from bottom to top of page. The bearing is taken at each setting and recorded *just above* the corresponding point in the note-book, or opposite a part of the line, rather than opposite the point. Ordinarily, the transitman takes the bearings of all fences and roads crossed by the line, finds the stations from the rear chainman, and records them in their proper place and direction on the right-hand page of the note-book. Section lines of the United States Land Surveys should be

observed in the same way. The transitman is next in authority to the locating engineer, and directs the work when the latter is not immediately present. The transitman, while moving from point to point, setting up, and ranging line, limits the speed of the entire party, and should waste no time.

22. The Head Chainman carries a "flag" and the forward end of the tape, which should be held level and firm with one hand, while the flag is moved into line with the other. He should always put himself nearly in line before receiving a signal from the transitman; plumbing may be done with the flag. When the point is found, the stakeman will set the stake. When a suitable place for a turning-point is reached, a signal should be given the transitman to that effect. A nail should be set in top of a "plug" at all turning-points. A proper understanding should be had with the transitman as to signals.

Signals from the Transitman.

A horizontal movement of the hand indicates that the rod should be moved as directed.

A swinging movement of the hand, "Plumb the rod as indicated."

A movement of both hands, or waving the handkerchief freely above the head, means "All right."

At long distances, a handkerchief can be seen to advantage; when snow is on the ground, something black is better.

Signals from the Head Chainman.

Setting the bottom of flag on the ground and waving the top, means "Give the line."

Raising the flag above the head and holding it horizontal with both hands: "Give line for a turning-point."

The "all right" signal is the same as from the transitman.

In all measurements less than 100 feet (or a full tape), the head chainman holds the end of the tape, leaving the reading of the measurement to the rear chainman.

The head chainman regulates the speed of the party during the time that the instrument is in place, and should keep alive all the time. The rear chainman will keep up as a matter of necessity.

23. The Stakeman carries, marks, and drives the stakes at the points indicated by the head chainman. The stakes should

be driven with the flat side towards the instrument, and marked on the front with the number of the station. Intermediate stakes should be marked with the number of the last station + the additional distance in feet and tenths, as 10 + 67.4. The stationing is not interrupted and taken up anew at each turning-point, but is continuous from beginning to end of the survey. At each turning-point a plug should be driven nearly flush with the ground, and a witness stake driven, in an inclined position, at a distance of about 15 inches from the plug, and at the side towards which the advance line deflects, and marked W and under it the station of the plug.

24. The Rear Chainman holds the rear end of the tape over the stake last set, but does not hold against the stake to loosen it. He calls "Chain" each time when the new stake is reached, being careful not to overstep the distance. He should stand beside the line (not on it) when measuring, and take pains not to obstruct the view of the transitman. He checks, and is responsible for the correct numbering of stakes, and for all distances less than 100 feet, as the head chainman always holds the end of the tape. The stations where the line crosses fences, roads, and streams should be set down in a small note-book, and reported to the transitman at the earliest convenient opportunity. The rear chainman is responsible for the tape.

25. The Back Flag holds the flag as a backsight at the point last occupied by the transit. The only signals necessary for him to understand from the transitman are "plumb the flag" and "all right." The flag should always be in position, and the transitman should not be delayed an instant. The back flag should be ready to come up the instant he receives the "all right" signal from the transitman. The duties are simple, but frequently are not well performed.

26. The Axeman cuts and clears through forest or brush. A good axeman should be able to keep the line well, so as to cut nothing unnecessary. In open country, he prepares the stakes ready for the stakeman or assists the locating engineer as *fore flag*.

27. The Leveler handles the level and generally keeps the notes, which may have the following form for the left-hand page. The right-hand page is for remarks and descriptions of turning-points and bench-marks. It is desirable that turning-

Station	+ S	H I	− S	Elevation
B.M.	4.67	104.67		100.00
0			5.7	99.0
1			6.9	97.8
2			3.4	101.3
T.P.	9.26	112.81	1.12	103.55
3			8.5	104.3

points should, where possible, be described, and that all bench-marks should be used as turning-points. Readings on turning-points should be recorded to hundredths or to thousandths of a foot, dependent upon the judgment of the Chief Engineer. Surface readings should be made to the nearest tenth, and elevations set down to nearest tenth only. A self-reading rod has advantages over a target rod for short sights. A target rod is possibly better for long sights and for turning-points. The "Philadelphia Rod" is both a target rod and a self-reading rod, and is thus well adapted for railroad use. Bench-marks should be taken at distances of from 1000 to 1500 feet, depending upon the country. All bench-marks, as soon as calculated, should be entered together on a special page near the end of the book. The leveler should test his level frequently to see that it is in adjustment. The leveler and rodman should together bring the notes to date every evening and plat the profile to correspond.

The profile of the preliminary line should show: —

a. Surface line (in black).
b. Grade line (in red).
c. Grade elevations at each change in grade (in red).
d. Rate of grade, per 100 (in red); rise +, fall −.
e. Station and deflection at each angle in the line (in black).
f. Notes of roads, ditches, streams, bridges, etc. (in black).

28. The Rodman carries the rod and holds it vertical upon the ground at each station and at such intermediate points as mark any important change of slope of the ground. The surface of streams and ponds should be taken when met, and at frequent intervals where possible, if they continue near the line.

Levels should also be taken of high-water marks wherever traces of these are visible. The rodman carries a small note-book in which he enters the rod readings at all turning-points. In country which is open, but not level, the transit party is liable to outrun the level party. In such cases greater speed will be secured by the use of two rodmen.

29. The Topographer is, or should be, one of the most valuable members of the party. In times past it has not always been found necessary to have a topographer, or if employed, his duty has been to sketch in the general features necessary to make an attractive map, and represent hills and buildings sufficiently well with reference to the line to show, in a general way, the reason for the location adopted. Sometimes the chief of the party has for this purpose taken the topography. At present the best practice favors the taking of accurate data by the topography party.

The topographer (with one or two assistants) should take the station and bearing (or angle) of every fence or street line crossed by the survey (unless taken by the transit party); also take measurements and bearings for platting all fences and buildings near enough to influence the position of the Location; also sketch, as well as may be, fences, buildings, and other topographical features of interest which are too remote to require exact location; and finally establish the position of contour lines, streams, and ponds, within limits such that the Location may be properly determined in the contoured map.

The work of locating contours is usually accomplished by the use of hand level and tape (distances carefully paced may, in many cases, be sufficiently accurate). The level party has determined the elevations of the ground at each "station" set by the transit party. These elevations are given the topographers to serve as bench marks for use in locating contours. It is customary to fix on the ground the points where the contours cross the center line, where they cross lines at right angles to the center line at each station, and occasionally additional points; then to sketch the contours by eye between these points. Cross section sheets in blocks or in book form are used for this purpose. The usual contour interval is 5 feet.

A point on a contour is found as follows. The topographer stands at the station stake; a measurement is taken, by tape

or rod, of the distance from the topographer's feet to his eye; this added to the surface height at center stake (as obtained from the level party) gives the "height of eye" above datum. The difference between this "height of eye" and the elevation of the contour gives the proper rod reading for fixing a point on the contour, and the rod is carried vertically along the ground until this reading is obtained. The point thus found is then located. The topographer uses this point, already fixed, as a turning point, finds anew his "height of eye," and proceeds to find a point on the next contour. It is more convenient at times to carry on the process in reverse order; that is, to hold the rod on the ground at the station, and let the topographer place himself where his feet are on the contour. The "height of eye" must be the distance from topographer's feet to eye added to the elevation of contour. The rod reading at the station will be the difference between this "height of eye" and the elevation of the ground at the station.

The hand level is somewhat lacking in precision, but by making a fresh start at each station as a bench mark, cumulative errors are avoided, and fair results secured by careful work. Instead of a hand level, some topographers use a clinometer, and take and record side slopes as a basis for contour lines.

Topography can be taken rapidly and well by stadia survey or by plane table. This is seldom done, as many engineers are not sufficiently familiar with their use. Much more accurate results may be reached by plane table, and a party of three, well skilled in plane table work, will accomplish more than a party of three with hand level.

30. Some engineers advocate making a general topographical survey of the route by stadia, instead of the survey above described. In this case no staking out by "stations" would be done. All points occupied by the transit should be marked by plugs properly referenced, which can be used to aid in marking the Location on the ground after it is determined on the contour map. This method has been used a number of times, and is claimed to give economical and satisfactory results; it is probable that it will have constantly increasing use in the future, and may prove the best method in a large share of cases.

CHAPTER III.

III. LOCATION SURVEY.

31. The Location Survey is the final fitting of the line to the ground. In Location, curves are used to connect the straight lines or "tangents," and the alignment is laid out complete, ready for construction.

The party is much the same as in the preliminary, and the duties substantially the same. More work devolves upon the transitman on account of the curves, and it is good practice to add a "note-keeper" to the party; he takes some of the transitman's work, and greater speed for the entire party is secured. More skill is useful in the head chainman in putting himself in position on curves. He can readily range himself on tangent. The form of notes will be shown later. The profile is the same, except that it shows, for alignment notes, the *P.C.* and *P.T.* of curves, and also the degree and central angle, and whether to the right or left.

It is well to connect frequently location stakes with preliminary stakes, when convenient, as a check on the work.

In making the location survey, two distinct methods are in use among engineers: —

32. First Method of Location. — Use preliminary survey and preliminary profile as guides in reading the country, and locate the line upon the ground. Experience will enable an engineer to get very satisfactory results in this way, in nearly all cases. The best engineers, in locating in this way, as a rule lay the tangents first, and connect the curves afterwards.

33. Second Method. — Use preliminary line, preliminary profile, and especially the contour lines on the preliminary map; make a paper location, and run this in on the ground. Some go so far as to give their locating engineer a complete set of notes to run by. This is going too far. It is sufficient to fix,

on the map, the location of tangents, and specify the degree of curve. The second method is much more desirable, but the first method has still some use. It is well accepted, among engineers, that no reversed curve should be used; 200 feet of tangent, at least, should intervene. Neither should any curve be very short, say less than 300 feet in length.

34. A most difficult matter is the laying of a long tangent, so that it shall be straight. Lack of perfect adjustment and construction of instrument will cause a "swing" in the tangent. The best way is to run for a distant foresight. Another way is to have the transit as well adjusted as possible, and even then change ends every time in reversing, so that errors shall not accumulate. It will be noticed that the preliminary is run in without curves because more economical in time; sometimes curves are run however, either because the line can be run closer to its proper position, or sometimes in order to allow of filing plans with the United States or separate States.

35. In Location, a single tangent often takes the place of a broken line in the preliminary, and it becomes important to determine the direction of the tangent with reference to some part of the broken line. This is readily done by finding the coördinates of any given point with reference to that part of the broken line assumed temporarily as a meridian. The course of each line is calculated, and the coördinates of any point thus found. It simplifies the calculation to use some part of the preliminary as an assumed meridian, rather than to use the actual bearings of the lines. The coördinates of two points on the proposed tangent allow the direction of the tangent to be determined with reference to any part of the preliminary. When the angles are small, an approximation sufficiently close will be secured, by assuming in all cases that the cosine of the angle is 1.000000 and that the sines are directly proportional to the angles themselves. In addition to this, take the distances at the nearest even foot, and the calculation becomes much simplified.

36. The located line, or "Location," as it is often called, is staked out ordinarily by center stakes which mark a succession of straight lines, connected by curves to which the straight lines are tangent. The straight lines are by general usage called **"Tangents."**

CHAPTER IV.

SIMPLE CURVES.

37. The curves most generally in use are circular curves, although parabolic and other curves are sometimes used. Circular curves may be classed as **Simple, Compound, Reversed,** or **Spiral.**

A Simple Curve is a circular arc, extending from one tangent to the next. The point where the curve leaves the first tangent is called the "*P. C.*," meaning the point of curvature, and the point where the curve joins the second tangent is called the "*P. T.*," meaning the point of tangency. The *P. C.* and *P. T.* are often called the Tangent Points. If the tangents be produced, they will meet in a point of intersection called the "**Vertex,**" *V.* The distance from the vertex to the *P. C.* or *P. T.* is called the "**Tangent Distance,**" *T.* The distance from the vertex to the curve (measured towards the center) is called the **External Distance,** *E.* The line joining the middle of the **Chord,** *C*, with the middle of the curve subtended by this chord, is called the **Middle Ordinate,** *M.* The radius of the curve is called the **Radius,** *R.* The angle of deflection between the tangents is called the **Intersection Angle,** *I.* The angle at the center subtended by a chord of 100 feet is called the **Degree of Curve,** *D.* A chord of less than 100 feet is called a **sub-chord,** *c*; its central angle a **sub-angle,** *d.*

38. The measurements on a curve are made:

(*a*) from *P. C.* by a sub-chord (sometimes a full chord of 100 ft.) to the next full station, then

(*b*) by chords of 100 feet each between full stations, and finally,

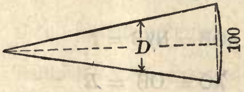
(*c*) from the last station on the curve, by a sub-chord (sometimes a full chord of 100 ft.) to *P. T.* The total distance from *P. C.* to *P. T.* *measured in this way*, is the **Length of Curve,** *L.*

39. The **Degree of Curve** is defined as the angle subtended by a *chord* of 100 feet, rather than by an *arc* of 100 feet.

Either assumption requires approximate methods either in calculations or measurements, if the convenient and customary methods are followed. On the merits of the question, it is best to accept the definition given, and the practice in this country is largely in harmony with this definition, which is adopted by the American Railway Engineering Association.

Outside of the United States a curve is generally designated by its Radius, R. In the United States for railroad purposes, a curve is generally designated by its Degree, D.

40. Problem. *Given R.*
Required D.



$$R \sin \tfrac{1}{2} D = \frac{100}{2}$$

$$\sin \tfrac{1}{2} D = \frac{50}{R}\,. \qquad (1)$$

41. Problem. *Given D.*
Required R.

$$R = \frac{50}{\sin \tfrac{1}{2} D} \qquad (2)$$

Example. *Given* $D = 1°$.

$$R_1 = \frac{50}{\sin \tfrac{1}{2} D} \qquad \begin{array}{rll} 50 & \log & 1.698970 \\ 0° \, 30' & \log \sin & 7.940842 \\ & \log & 3.758128 \end{array}$$

$$R_1 = 5729.6$$

42. Problem. *Given* R_1 (radius of 1° curve) or D_1.
Required R_a (radius of any given curve of degree $= D_a$).

$$R_1 = \frac{50}{\sin \tfrac{1}{2} D_1} \qquad R_a = \frac{50}{\sin \tfrac{1}{2} D_a}$$

$$\frac{R_a}{R_1} = \frac{\sin \tfrac{1}{2} D_1}{\sin \tfrac{1}{2} D_a} \qquad R_a = R_1 \frac{\sin \tfrac{1}{2} D_1}{\sin \tfrac{1}{2} D_a} \qquad (3)$$

In the case of small angles, the angles are proportional to the sines (approximately),

$$R_a = R_1 \frac{\tfrac{1}{2} D_1}{\tfrac{1}{2} D_a}\,; \quad R_a = R_1 \frac{D_1}{D_a} \qquad (3\,A)$$

But $R_1 = 5730$ to nearest foot,

$$R_a = \frac{5730}{D_a} \text{ (approx.)} \qquad (4)$$

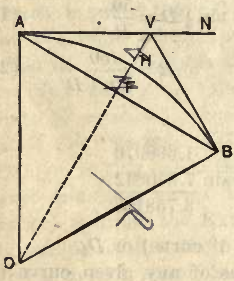
Example. $R_{10} = 573.7$ by (3), or by Allen's Table I.
$= 573.0$ by (4) (approx.)

Some engineers use shorter chords for sharp curves, as 1° to 7°, 100 ft. ; 8° to 15°, 50 ft.; 16° to 20°, 25 ft.

Values of R and D are readily convertible. For this purpose, use Table I., Allen [rather than formula (1) or (2)], when accurate results are required. In problems later, where either R or D is given, both will, in general, be assumed to be given. Approximate values can be found without tables by (4). The radius of a 1° curve = 5730 should be remembered. Precise results are, in general, necessary.

43. Problem. *Given I, also R or D.*

Required T.



$$AOB = NVB = I$$

$$AO = OB = R$$

$$AV = VB = T$$

$$T = R \tan \tfrac{1}{2} I \qquad (5)$$

Example. *Given* $D = 9$; $I = 60° 48'$.

Required T_9.

Table I., $R_9 \log = 2.804327$

$30° 24' \log \tan = 9.768414$

$T_9 = 373.9 \log 2.572741$

Note that log R_9 is taken directly from Table I.

44. Approximate Method.

$$T_1 = R_1 \tan \tfrac{1}{2} I; \quad T_a = R_a \tan \tfrac{1}{2} I$$

$$\frac{T_a}{T_1} = \frac{R_a}{R_1} = \frac{D_1}{D_a} = \frac{1}{D_a} \text{ (approx.)}$$

$$T_a = \frac{T_1}{D_a} \text{ (approx.)} \qquad (6)$$

Table III., Allen, gives values of T_1 for various values of I.

Table IV., Allen, gives a correction to be added *after* dividing by D_a.

Example. As before. *Given* $D = 9$; $I = 60° 48'$.

Required T_9.

Table III.,	T_1 60° 48′	=	3361.6(9
	T_9	=	373.51 (approx.)
Table IV., correction, 9° and 61°		=	.38
	T_9	=	373.9 (exact)

the same as before

45. Problem. *Given I, also R or D.*

Required E.

Using previous figure, $\mathsf{VH} = E = R \text{ exsec } \frac{1}{2} I$ (7)

Table XXXIII. shows definition of exsecant.
Table XIX. gives natural exsec.
Table XV. gives logarithmic exsec.
Approximate Method.

By method used for (6), $E_a = \frac{E_1}{D_a}$ (approx.) (8)

Table V. gives values for E_1.

46. Problem. *Given I, also R or D.*

Required M.

$$\mathsf{FH} = M = R \text{ vers } \tfrac{1}{2} I \tag{9}$$

Table XXXIII. shows definition of versine.

Table XIX. gives natural vers.
Table XV. gives logarithmic vers.
Table II. gives certain middle ordinates.

47. Problem. *Given I, also R or D.*

Required chord AB = *C.*

$$C = 2\, R \sin \tfrac{1}{2} I \tag{10}$$

Table VIII. gives values for certain long chords.

48. Transposing, we find additional formulas, as follows:

from (5) $R = T \cot \frac{1}{2} I$ (11)

(7) $R = \dfrac{E}{\operatorname{exsec} \frac{1}{2} I}$ (12)

(9) $R = \dfrac{M}{\operatorname{vers} \frac{1}{2} I}$ (13)

(10) $R = \dfrac{C}{2 \sin \frac{1}{2} I}$ (14)

(4) $D_a = \dfrac{5730}{R_a}$ (approx.) (15)

(6) $D_a = \dfrac{T_1}{T_a}$ (approx.) (16)

(8) $D_a = \dfrac{E_1}{E_a}$ (approx.) (17)

49. Problem. *Given sub-angle d, also R or D.*
Required sub-chord c.

$$c = 2 R \sin \tfrac{1}{2} d \qquad (18)$$

Approximate Method.

$$100 = 2 R \sin \tfrac{1}{2} D$$

$$\frac{c}{100} = \frac{\sin \frac{1}{2} d}{\sin \frac{1}{2} D} = \frac{d}{D} \text{ (approx.)} \qquad (19)$$

The precise formula is seldom if ever used.

50. Problem. *Given sub-chord c, also R or D.*
Required sub-angle d.

$$d = \frac{cD}{100} \qquad (20)$$

The value $\frac{d}{2}$ is more frequently needed and

$$\frac{d}{2} = \frac{c}{100} \frac{D}{2} \qquad (21)$$

A modification of this formula is as follows:

$$\frac{d}{2} = \frac{cD}{200}$$

for $D = 1$

$$\frac{d}{2} = c\frac{60'}{200} = c \times 0.3'$$

for any value D_a

$$\frac{d}{2} = c \times 0.3' \times D_a \text{ (result in minutes)} \qquad (22)$$

This gives a very simple and rapid method of finding the value of $\frac{d}{2}$ in minutes, and the formula should be remembered.

Example. *Given sub-chord* $= 63.7$. $D = 6° 30'$.
Required sub-angle $d \left(or\ \frac{d}{2}\right)$.

I. By (20)

$$\begin{array}{r} 63.7 \\ \underline{6.5} = D \\ 3185 \\ \underline{3822} \\ 414.05 \\ \\ 4°.14 \\ \underline{60'} \\ d = 4° 08' \\ \frac{d}{2} = 2° 04' \end{array}$$

II. By (21)

$$\begin{array}{r} 63.7 \\ \underline{3.25} = \frac{D}{2} \\ 3185 \\ 1274 \\ \underline{1911} \\ 207.025 \\ \\ 2°.07 \\ \underline{60'} \\ \frac{d}{2} = 2° 04' \end{array}$$

III. By (22)

$$\begin{array}{r} 63.7 \\ \underline{0.3} \\ 19.11 \\ \underline{6.5} = D \\ 9555 \\ \underline{11466} \\ 124.215 \text{ minutes} \\ \\ \frac{d}{2} = 2° 04' \end{array}$$

Method III. is often preferable to I. or II.

51. Problem. *Given I and D.*
Required L.

The "Length of Curve" L is the distance around the curve, measured as stated in § 38, or $L = c_1 + 100\,n + c_2$.

(a) When the *P.C.* is at a *full station*, D will be contained in I a certain number of times n, and there will remain a sub-angle d_2 subtended by its chord c_2, and $L = 100\,n + c_2$.

$$\frac{I}{D} = n + \frac{d_2}{D} = \quad n + \frac{c_2}{100} \text{ (approx.)}$$

$$100\,\frac{I}{D} \qquad = 100\,n + c_2 = L \text{ (approx.)}$$

(b) When the *P.C.* is at a *sub-station* and *P.T.* at a *full station*, the same reasoning holds, and

$$L = 100\frac{I}{D} \text{ (approx.)}$$

(c) When both *P.C.* and *P.T.* are at *sub-stations*, the same formula holds

$$L = 100\frac{I}{D} \text{ (approx.)} \tag{23}$$

Transposing,
$$I = \frac{LD}{100} \text{ (approx.)} \tag{24}$$

$$D = \frac{100\,I}{L} \tag{25}$$

These formulas (23)(24)(25), though approximate, are the formulas in common use.

Example. *Given* 7° *curve.* $I = 39°\,37'$. *Required L.*

$$\begin{aligned} I &= \ \ 39°\,37 \\ D &= 7)\underline{39.6167°} \\ &\qquad 5.6595 + \qquad\qquad L = 566.0 \end{aligned}$$

Example. *Given D and L. Required I.*
Given 8° *curve.*
also, $P.T. = 93 + 70.1$

$$\begin{aligned} P.C. &= \underline{86 + 49.3} \\ L &= \ 7 \quad 20.8 \\ D &= \underline{\qquad\quad 8} \\ &\qquad 57.664 \\ &\qquad\quad \underline{60'} \\ &\qquad 39.84 \qquad\qquad I = 57°\,40' \end{aligned}$$

52. Field-work of finding *P. C.* and *P. T.*

In running in the line, it is common practice to continue the stationing as far as V, to set a plug and mark a witness stake with the station of V as thus obtained. The angle I is then measured and "repeated" as a "check."

Having given I only, an infinite number of curves could be used. It is, therefore, necessary to assume additional data to determine what curve to use. It is common to proceed as follows:

(*a*) Assume either (1) D directly.
(2) E and calculate D.
(3) T and calculate D.

It is often difficult to determine off-hand what degree of curve will best fit the ground. Frequently the value of E_a can be readily determined on the ground. The determination of D from E_a is readily made, using the approximate formula $D_a = \frac{E_1}{E_a}$. Similarly, we may be limited to a given (or ascertainable) value of T_a, and from this readily find $D_a = \frac{T_1}{T_a}$.

This process is to determine what value of D_a will fit the ground, and it is convenient, generally, to use the degree or half degree nearest to that calculated. (Some engineers use $1° 40' = 100'$ and $3° 20' = 200'$, etc., rather than $1° 30'$ or $3° 30'$, etc.)

When the D_a is thus determined, all computations must be strictly based on this value of D_a.

(*b*) From the data finally adopted T is *calculated anew.*

(*c*) The instrument still being at V, the *P. T.* is set by laying off T. It is economical to set *P. T.* before *P. C.*

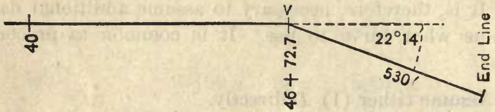
(*d*) The station of *P. C.* is calculated and *P. C.* set from the nearest station stake (or by measuring back from V).

(*e*) The length of curve L is calculated, and station of *P. T.* thus determined (not by adding T to station of V).

Whether D, E, or T shall be assumed depends upon the special requirements in each case. Curves are often run out from *P. C.* without finding or using V, but the best engineering usage seems to be in favor of setting V, whenever this is at all practicable, and from this finding the *P. C.* and *P. T.*

53. Example. *Given a line, as shown in sketch. Required a Simple Curve to connect the Tangents.*

P.T. is to be at least 300 ft. from end of line.
Use smallest degree or half degree consistent with this.
Find degree of curve and stations of *P.C.* and *P.T.*



		530	
		300	
Table III. $22° 14'$	$T_1 = 1125.8$	$\overline{230} = T$ (approx.)	
	92	4.9 −	use 5° curve
$T_1 = \underline{1125.8}(5°$	$\overline{205}$		
225.16			$V = 46 + 72.7$
Table IV. corr. .07	$I = 22° 14'$		$T = 2 + 25.2$
$T =$ 225.2	$= \underline{22°.2333}(5°$		$P.C. = 44 + 47.5$
	$L =$ 444.7		$L = 4 + 44.7$
			$P.T. = 48 + 92.2$

It will be noticed that the station of the *P.T.* is found by adding L to the stations of the *P.C.* (*not* by adding T to the station of V).

Similarly *Given* $E = 17$ ft.

Table V. $22° 14'$	$E_1 = 109.6$	$17 = E$	
	102	6.4 +	use 6° 30′ curve
$T_1 = \underline{1125.8}(6.5$	$\overline{76}$		
173.20	$I = \underline{22°.2333}(6.5$		$V = 46 + 72.7$
corr. .09	$L =$ 342.1		$T = 1 + 73.3$
$T =$ 173.3			$P.C. = 44 + 99.4$
			$L = 3 + 42.1$
			$P.T. = 48 + 41.5$

Under the conditions prescribed above, when T is given, the degree, or half degree, next larger must be used, in order to secure *at least* the required distance (to end of line above).

When E is given, the nearest half degree is generally used.

54. Method of **Deflection Angles.**

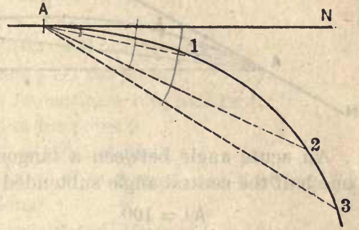
If at any point on an existing curve a tangent to the curve be taken, the angles from the tangent to any given points on the curve may be measured, and the angles thus found may be called **Total Deflections** to those points (as NA1, NA2, NA3).

In laying out successive points upon a straight line (as on a "Tangent"), each point is generally fixed by

(*a*) measurement from the preceding point and

(*b*) line;

the line on a tangent will be the same for all points.



Similarly, in laying out a curve, successive points may be fixed by

(*a*) measurement from the preceding point and

(*b*) line;

the line in this case, for the curve, will be that found by using the *total deflection* calculated for each point. In the figure preceding, the chord distance A1 and the total deflection NA1 fix point 1; the chord distance 1–2 and total deflection NA2 fix point 2; and 2–3 and NA3 fix 3. A curve can be conveniently laid out by this method if the proper total deflections can be readily computed.

55. The method of "Deflection Angles" is well adapted to surveying an existing curve; it is also well adapted to laying out any curve, provided only that it is possible to readily determine

(*a*) the "Total Deflection Angles" and

(*b*) the chord lengths that go with them.

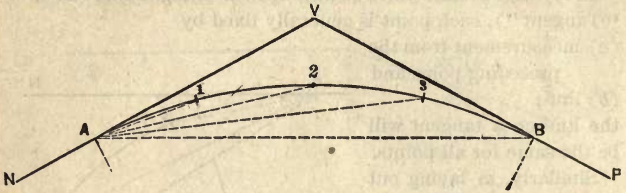
In the case of "*Simple Curves*," the "total deflections" can be readily computed, and the method of "deflection angles" is therefore well adapted to laying them out.

In the case of "spiral" or "transition" curves, tables have commonly been computed, so that the angles and chords are taken from the tables. Any curve which has been surveyed by this method can be restaked on the ground by using the deflection angles and chords measured and recorded.

56. Problem. To find the *Total Deflections* for a Simple Curve having *given* the *Degree.*

I. *When the curve begins and ends at even stations.*

The distance from station to station is 100 feet. The deflection angles are required.



An acute angle between a tangent and a chord is equal to one half the central angle subtended by that chord

$$A\,1 = 100 \qquad V\,A\,1 = \tfrac{1}{2} D$$

The acute angle between two chords which has its vertex in the circumference is equal to one half the arc included between those chords.

$$1 - 2 = 100 \text{ and } 1\,A\,2 = \tfrac{1}{2} D \quad \text{Similarly,}$$

$$2 - 3 = 100 \text{ and } 2\,A\,3 = \tfrac{1}{2} D$$

$$3 - B = 100 \text{ and } 3\,A\,B = \tfrac{1}{2} D$$

This angle $\frac{1}{2} D$ is called by Henck and Searles the **Deflection Angle**, and will be so called here. Shunk and Trautwine call it the "*Tangential Angle*." The weight of engineering opinion appears to be largely in favor of the "Deflection Angle."

The "*Total Deflections*" will be as follows:

$$V\,A\,1 = \tfrac{1}{2} D$$

$$V\,A\,2 = V\,A\,1 + \tfrac{1}{2} D$$

$$V\,A\,3 = V\,A\,2 + \tfrac{1}{2} D$$

VAB will be found by successive increments of $\frac{1}{2} D$.

$VAB = VBA = \frac{1}{2} I$. This furnishes a "check" on the computation.

II. *When the curve begins and ends with a sub-chord.*

$$V\,A\,1 = \tfrac{1}{2} d$$

$$V\,A\,2 = V\,A\,1 + \tfrac{1}{2} D$$

$$V\,A\,3 = V\,A\,2 + \tfrac{1}{2} D$$

VAB is found by adding $\frac{1}{2} d_2$ to previous "total deflection."

$VAB = VBA = \frac{1}{2} I$. This furnishes "check." The total deflections should be calculated by successive increments; the final "check" upon $\frac{1}{2} I$ then checks all the intermediate total deflections. The example on next page will illustrate this.

57. Field-work of laying out a simple curve having given the position and station of *P.C.* and *P.T.*

(*a*) Set the transit at *P.C.* (A).

(*b*) Set the vernier at 0.

(*c*) Set cross hairs on V (or on N and reverse).

(*d*) Set off $\frac{1}{2} d_1$(sometimes $\frac{1}{2} D$) for point 1.

(*e*) Measure distance c_1 (sometimes 100) and fix 1.

(*f*) Set off total deflection for point 2.

(*g*) Measure distance 1–2 = 100 and fix 2, etc.

(*h*) When total deflection to B is figured, see that it $= \frac{1}{2} I$, thus "checking" calculations.

(*i*) See that the proper calculated distance c_2 and the total deflection to B agree with the actual measurements on the ground, checking the field-work.

(*k*) Move transit to *P.T.* (B).

(*l*) Turn vernier back to 0, and *beyond* 0 *to* $\frac{1}{2} I$.

(*m*) Sight on A.

(*n*) Turn vernier to 0.

(*o*) Sight towards V (or reverse and sight towards P), and see that the line checks on V or P.

It should be observed that three "checks" on the work are obtained.

1. The calculation of the total deflections is checked if total deflection to $B = \frac{1}{2} I$.

2. The chaining is checked if the final sub-chord measured on the ground = calculated distance.

3. The transit work is checked if the total deflection to B brings the line accurately on B.

The check in 1 is effective only when the total deflection for each point is found by adding the proper angle to that for the preceding point.

The check in 3 assures the general accuracy of the transit work, but does not prevent an error in laying off the total deflection at an intermediate point on the curve.

58. Example. *Given Notes of Curve*

$$\begin{array}{ll} P.T. & 13+45.0 \\ P.C. & 10+74.0 \end{array} \qquad 6^\circ \text{ curve } \mathbf{L}$$

Required the "total deflections"

$$\begin{array}{rrrl}
\text{to sta. } 11 \quad c_1 = & 26 & & \\
 & \underline{.3} & & \\
 & 7.8 & & \\
 & \underline{6^\circ} & & \\
\frac{d_1}{2} = & 46.8 & = 0^\circ 47' & \text{to } 11 \\
 & & \underline{3^\circ} & \\
c_2 = & 45 & 3^\circ 47' & \text{to } 12 \\
 & \underline{.3} & \underline{3^\circ} & \\
 & 13.5 & 6^\circ 47' & \text{to } 13 \\
 & \underline{6^\circ} & & \\
\frac{d_2}{2} = & 81.0 & = \underline{1^\circ 21'} & \\
 & & 8^\circ 08' & \text{to } 13+45
\end{array}$$

$$\begin{array}{lll}
13+45.0 & & \\
\underline{10+74.0} & & \\
2 \quad 71.0 = L & & \\
\quad \underline{6^\circ} & & \\
16.26 & 16^\circ & \\
\quad \underline{60'} & & \\
15.6' & \underline{\quad 16'} & \\
 & 16^\circ 16' = I & \\
 & 8^\circ 08' = \tfrac{1}{2} I \text{ "check"} &
\end{array}$$

59. Caution.

If a curve of nearly $180^\circ = I$ is to be laid out from A, it is evident that it would be difficult or impossible to set the last point accurately, as the "intersection" would be bad. It is undesirable to use a total deflection greater than 30°.

It may be impossible to see the entire curve from the *P.C.* at A.

It will, therefore, frequently happen that from one cause or another the entire curve cannot be laid out from the *P.C.*, and it will be necessary to use a modification of the method described above.

60. Field-work. *When the entire curve cannot be laid out from the P. C.*

First Method.

(a) Lay out curve as far as C, as before.

(b) Set transit point at some convenient point, as C (even station preferably) and move transit to C.

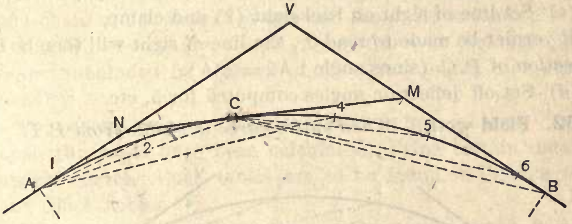
(c) Turn vernier back to 0°, *and beyond* 0° by the value of angle VAC.

(d) Sight on A.

(e) Turn vernier to 0°. See that transit line is on auxiliary tangent NCM (VAC = NCA being measured by $\frac{1}{2}$ arc AC).

(f) Set off new deflection angle ($\frac{1}{2}d$ or $\frac{1}{2}D$).

(g) Set point 4, and proceed as in ordinary cases.



Second Method.

(a) Set point C as before, and move transit to C.

(b) Set vernier at 0° and sight on A.

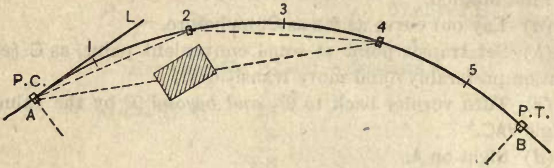
(c) Set off the proper "total deflection" for the point 4 = VA 4 NCA + MC 4 = VA 4, each measured by $\frac{1}{2}$ arc AC 4.

(d) Reverse transit and set point 4.

(e) Set off and use the proper "total deflections" for the remaining points.

The second method is in some respects more simple, as the notes and calculations, and also setting off angles, are the same as if no additional setting were made. By the first method the deflection angles to be laid off will, in general, be even minutes, often degrees or half degrees, and are thus easier to lay off. It is a matter of personal choice which of the two methods shall be used. It will be disastrous to attempt an incorrect combination of parts of the two methods.

61. Field-work. *When the transit is in the curve, and the P.C. is not visible.*



(*a*) Compute deflection angles, *P.C.* to *P.T.*; check on $\frac{I}{2}$ (same as in § 56).

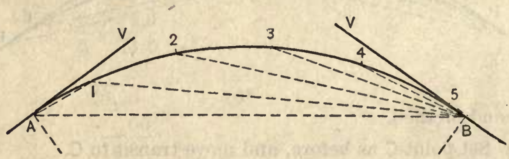
(*b*) Set vernier at deflection angle computed for point (*e.g.* 2) used as backsight.

(*c*) Set line of sight on backsight (2) and clamp.

If vernier be made to read 0°, the line of sight will then be in direction of *P.C.* (since angle $\angle A2 = 24A$).

(*d*) Set off deflection angles computed for 5, etc.

62. Field-work. *When entire curve is visible from P.T.*



(*a*) Compute deflection angles, *P.C.* to *P.T.*; check on $\frac{I}{2}$ (same as in § 56).

(*b*) Set transit at *P.T.* with vernier at 0° and sight on *P.C.*

(*c*) Set off computed angles for 1, 2, 3, 4, 5.

(*d*) Set off $\frac{I}{2}$ and sight at V for check on transit work.

This method is preferable to that given in § 57. It saves the transit setting at *P.C.* The long sights are taken first, before errors of chaining have accumulated and before the transit has settled or warped in the sunlight. The last point on curve is set at a small angle with the tangent, so that the intersection is good and any accumulated errors of chaining will not much affect the line. The method is already accepted practice.

63. Metric Curves.

In Railroad Location under the "**Metric System**" a chain of 100 meters is too long, and a chain of 10 meters is too short. Some engineers have used the 30-meter chain, some the 25-meter chain, but lately the 20-meter chain has been generally adopted as the most satisfactory. Under this system a "*Station*" is 10 meters. Ordinarily, every second station only is set, and these are marked Sta. O, Sta. 2, Sta. 4, etc. On curves, *chords of 20 meters* are used. Usage among engineers varies as to what is meant by the *Degree of Curve* under the metric system. There are two distinct systems used, as shown below.

I. The *Degree of Curve* is the *angle at the center* subtended by a chord of 1 chain of *20 meters.*

II. The *Degree of Curve* is the *deflection angle* for a chord of 1 chain of *20 meters* (or one half the angle at the center).

II. Or, very closely, the *Degree of Curve* is the *angle at the center* subtended by a chord of *10 meters* (equal to 1 station length).

For several reasons the latter system is favored here. Tables upon this basis have been calculated, giving certain data for metric curves. Such tables are to be found in Allen's Field and Office Tables.

In many countries where the metric system is used, it is not customary to use the *Degree of Curve*, as indicated here. In Mexico, where the metric system is adopted as the only legal standard, very many of the railroads have been built by companies incorporated in this country, and under the direction of engineers trained here. The usage indicated above has been the result of these conditions. If the metric system shall in the future become the only legal system in the United States, as now seems possible, one of the systems outlined above will probably prevail.

In foreign countries where the *Degree of Curve* is not used, it is customary, as previously stated, to designate the curve by its radius R, and to use even figures, as a radius of 1000 feet, or 2000 feet, or 1000 meters, or 2000 meters. As the radius is seldom measured on the ground, the only convenience in even figures is in platting, while there is a constantly recurring inconvenience in laying off the angles.

64. Form of Transit Book (left-hand page).

(Date)
(Names of Party)

Station	Points	Descrip. of Curve	Total Deflect.	Observed Course
114				
113				
112				
111				
110				
109				N 46° 00′ E
	⊙+ 90.0 *P. T.*		11° 15′	
108			9° 00′	
		$R = 1146.3$		
107		$L = 450.0$	6° 30′	
	⊙+ 68.0 *V*	$T = 228.0$		
106			4° 00′	
		$I = 22° 30′$		
105			1° 30′	
	⊙+ 40.0 *P.C.*	5° Right		
104				
103				
102				
101				
100				
99				N 23° 15′ E
98				

V is not a point on the curve. Nevertheless, it is customary to record the station found by chaining along the tangent.

The right-hand page is used for survey notes of crossings of fences and various similar data. It seems unnecessary to show a sample here.

65. Circular Arcs. For general railroad work, the Length of Curve is the distance measured by a series of chords as defined in § 38 and § 51. For certain purposes, largely outside of railroad work, the actual length of arc is required. Where the line of a street is curved, the length of the side line of street, the property line of a lot or estate, may be required. Furthermore, both in railroad and street railway work, actual lengths of rails are sometimes required.

Problem. *Given the Central Angle I and Radius R. Required the Length of Arc.*

Table XX., p. 205, Allen, gives lengths of circular arcs for radius $= 1$. The values for degrees, minutes and seconds are added; the sum multiplied by R is the required length of arc.

Example. *Given* $I = 18° 43' 29''$; $R = 600$.
Required Length of Arc and Deflection Angles.

$18°$	0.3141593
$43'$	0.0125082
$29''$	0.0001406
	0.3268081
R	600
Length of Arc = 196.08	196.08486

Where a series of points are to be set on the circular arc, there are several methods available, each of which has some desirable features.

I. (*a*) *Divide the entire arc length into an equal number of parts.*

(*b*) Compute the deflection angles to correspond.

(*c*) Compute the chord lengths to correspond.

In the example above, if 4 intermediate points are to be set on the arc, the length of arc will be divided by 5; the final deflection angle will be $\frac{1}{2} I$; and the first deflection angle, i_1, will be $\frac{1}{2} I \div 5$.

$5)\underline{196.08}$ = Length of Arc
39.216

$$I = 18° 43' 29''$$
$$\tfrac{1}{2} I = \underline{9° 21' 44''}(5$$
$$i_1 = 1° 52' 21''$$

The deflection angles will be (to nearest ½ minute) 1° 52′ 30″; 3° 44′ 30″; 5° 37′; 7° 29′ 30″; 9° 21′ 30″. For chaining, the length of chord is necessary and may be computed by formula (10). Where the radius is large, natural sines may not give satisfactory results, and it may be necessary to use the auxiliary tables of log. sines.

A simpler method is to use Allen's Table XX., A, which gives for $R = 1$ the difference between arc and chord for various central angles.

For central angle 3° 45′	diff.	=	0.000012	Table XX., A.
	R	=	600	
			0.007	
	Arc	=	39.216	
	Chord	=	39.209	

The *P.T.* of the circular arc should be set with the required precision by long chord from *P.C.* and the several chords measured with a degree of precision sufficient to secure a "check" against material error.

II. (*a*) *Use a series of equal chords of convenient length, followed by a sub-chord to the P.T.*

(*b*) Compute deflection angles to correspond.

(*c*) Compute arc lengths to correspond.

(*d*) Compute sub-chord length.

Example. *Given as before* $I = 18° 43' 29''$; $R = 600$.

Take chord length of 40 ft.

Let i_1 = deflection angle for chord of 40 ft.

Then $\sin i_1 = \frac{20}{600}$ $\qquad i_1 = 1° 54' 37''$

and corresponding central angle $d_1 = 3° 49' 14''$.

For central angle 3° 49′	diff. = 0.000012	Table XX., A.	
	R = 600		
	0.007		
	arc = 40.007		
4 lengths of arc	= 160.028		
entire arc	= 196.085	from p. 37	
sub-arc	= 36.057	for $R = 600$	
36.057 ÷ 600	= 0.060095	= sub-arc for $R = 1$	

From p. 38,		0.0600950 = sub-arc for $R = 1$
Table XX.,	$3°$	0.0523599
		0.0077351
	$26'$	0.0075631
		0.0001720
	$35''$	0.0001697

For central angle $3° 27'$ diff. = 0.000009

$R =$ 600

0.005

sub-arc = 36.057

sub-chord = 36.052

III. (*a*) *Use uniform deflection angles to some convenient even minute, except for final sub-chord.*

(*b*) Compute chord lengths to correspond.

(*c*) Compute arc lengths to correspond.

Example. *Given as before.* $I = 18° 43' 29''$ $R = 600$

For 5 equal arcs $i_1 = 1° 52' 21''$

Assume $i_1 = 2° 00'$; then $2\,i_1 = 4° 00' =$ central angle.

For central angle $4°$ diff. = 0.000014 Table XX., A

$R =$ 600

0.008

Chord length for $4° = 2 \times 600 \times \sin 2° =$ 41.880

arc length = 41.888

$I = 18° 43' 29''$

$4 \times$ central angle $4°$ $= 16°$

final sub-angle d_2 $= 2° 43' 29''$

For central angle $2° 43'$ diff. = 0.000004 Table XX., A.

$R =$ 600

0.002

For central angle $2° 43' 29''$ arc = 0.0475554 Table XX.

$R =$ 600

28.53324

diff. = 0.002

final sub-chord = 28.531

A convenient formula for the difference between chord and arc is the following, which though approximate, is essentially correct when the value of the chord c is not large in comparison with R.

Let l = length of arc

c = length of corresponding chord.

Then $l - c = \frac{c^3}{24 R^2} = \frac{l^3}{24 R^2}$ (both approximate).

For such values as $c = 100$ or $c = 50$ and $R = 1000$ or $R = 2000$, the computation is at once simple.

For other values, the computations are conveniently made on a slide rule.

No proof of this formula is given here. It may readily be proved along the lines of § 188, p. 119, making use of formula (26), p. 42, as a formula of the circle.

In a curved street, it is not uncommon to describe the alignment by giving the radius, R, of the center line, and also the distance (or stationing) along the center line measured along the arc (rather than by a series of chords as in railroad work).

It is also necessary to know the lengths l_l along the property line on the outside of the curve, and the length l_s along the inside line.

Let A = central angle subtending part or whole of a curve.

l_c = corresponding length on center line

l_l = " " " outside line

l_s = " " " inside line

w_l = width from center to outside line

w_s = " " " " inside line.

So that $w_l + w_s$ = total width of street.

Then

$$l_c = R \text{ angle } A$$

$$l_l = (R + w_l) \text{ angle } A$$

$$l_s = (R - w_s) \text{ angle } A$$

$$l_l - l_c = w_l \text{ angle } A$$

$$l_c - l_s = w_s \text{ angle } A$$

The values of w_l and w_s are usually not large, and commonly even numbers. The computations of differences, therefore, are more simply made than computations of total values.

This is true whether A subtends an arc of 100 ft., or a sub arc, or the full arc from *P. C.* to *P. T.*

Similarly for any chord on the center line

$$c_c = 2\,R \sin \tfrac{1}{2}\,A$$

$$c_l = 2(R + w_l) \sin \tfrac{1}{2}\,A$$

$$c_s = 2(R - w_s) \sin \tfrac{1}{2}\,A$$

$$c_l - c_c = 2\,w_l \sin \tfrac{1}{2}\,A$$

$$c_c - c_s = 2\,w_s \sin \tfrac{1}{2}\,A$$

Also
$$\frac{c_l}{c_c} = \frac{R + w_c}{R}$$

$$\frac{c_s}{c_c} = \frac{R - w_s}{R}$$

Sometimes one, sometimes the other, set of formulas will prove more convenient.

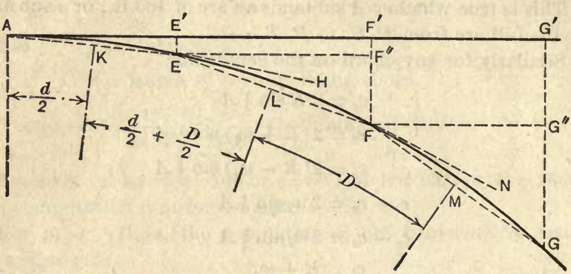
Where there are many points to be set, each side line, as well as the center, should be set by transit by deflection angles.

The following table shows the necessary data and a convenient form of notes.

Station	Description of Curve	Deflection Angles	Chords			Arcs
			Left	Center	Right	
+ 99.38 *P. T.*	To Right	19° 42′ 20″	25.18	21.71	18.24	21.72
+ 77.66	$R = 250$ $I = 39° 24' 40''$	17° 13′ 00″	58.00	50.00	42.00	50.08
9 + 27.58	$l_c = 171.96$	11° 28′ 40″	58.00	50.00	42.00	50.08
+ 77.50	$T = 89.54$ $w_l = 40.00$	5° 44′ 20″	58.00	50.00	42.00	50.08
8 + 27.42 *P. C.*	$w_s = 40.00$					

When the curve is short, and a few points only need be set on the outside and inside lines, these points may be set by fixing distance by the proper chord lengths, and line by measuring w_l or w_s from the appropriate point on the center line.

66. Problem. *Given D and stations of $P.C.$ and $P.T.$ Required* to lay out the curve by the method of **Offsets from the Tangent.**



Let AG′ be tangent to curve AG

Find $\quad E'AE = \frac{1}{2}\,d \qquad\qquad = \alpha_1$

When $AE = 100$, then $\frac{1}{2}\,d$ becomes $\frac{1}{2}\,D$.

$$F''EF = d + \tfrac{1}{2}\,D \qquad = \alpha_2$$

$$G''FG = d + D + \tfrac{1}{2}\,D = \alpha_3, \text{ etc.}$$

Draw EH tangent at E.

Also FN tangent at F.

The α for each chord is found by taking the *central* angle to the *beginning* of the chord plus the *deflection* angle for the chord.

$$\alpha_2 = F''EH + HEF$$
$$= d + \tfrac{1}{2}\,D$$
$$\alpha_3 = G''FN + NFG$$
$$= d + D + \tfrac{1}{2}\,D$$

$AE' = c_i \cos \alpha_1 \qquad EE' = c_i \sin \alpha_1$

$EF'' = 100 \cos \alpha_2 \qquad FF'' = 100 \sin \alpha_2$

$FG'' = 100 \cos \alpha_3 \qquad GG'' = 100 \sin \alpha_3$

$FF' = EE' + FF''$

$GG' = FF' + GG''$, etc.

For the computations indicated above, always use natural sines and cosines.

For a check, $\quad AG' = R \sin AOG$

$GG' = R \text{ vers } AOG$

where O is at center of curve.

For the computations immediately above, use log sines and versines.

These "check" computations involve the radius (or degree) and the central angle; the previous computations involve the use of c also; since the formula

$$d=\frac{cD}{100} \tag{20}$$

is an approximate formula, perfect precision in the "check" cannot be expected.

If a "check" perfectly precise is required, use formula (18) $c = 2\,R \sin \frac{1}{2}\,d$ instead of formula (20) and carry all intermediate work to the necessary degree of precision.

This method of Offsets from the Tangent is a precise method, and allows of any desired degree of precision in field-work.

Another method of finding the angles α_1, α_2, α_3, etc., is by drawing perpendiculars to the chords at K, L, and M.

Then
$$\begin{aligned}\alpha_1 &= \tfrac{1}{2}\,d\\ \alpha_2 &= \alpha_1 + \tfrac{1}{2}\,d + \tfrac{1}{2}\,D\\ &= d + \tfrac{1}{2}\,D \text{ (as before)}\\ \alpha_3 &= \alpha_2 + D \text{ etc.}\end{aligned}$$

Each α being found by adding an increment to the previous value of α.

Also $\alpha_3 = \text{AOG} - \frac{1}{2}\,D$

which gives a "check" on all values of α computed.

If AE, EF, FG, are parts of a compound curve, the same general methods are applicable, except that the checks of R sin AOG and R vers AOG are not then available.

67. Field-work.

(*a*) Calculate AE′, E′F′, F′G′; also EE′, FF′, GG′

(*b*) Set E′, F′, G′, by measurements AE′, E′F′, F′G′.

(*c*) Set E by distance AE (c_i) and EE′.

(*d*) Set F " " EF (100) and FF′.

(*e*) Set G " " FG (100) and GG′.

68. Problem. *Given D and the stations of P.C. and P.T. Required to lay out the curve by the method of* **Deflection Distances.**

When the curve begins and ends at even stations.

In the curve AB, let

AN be a tangent

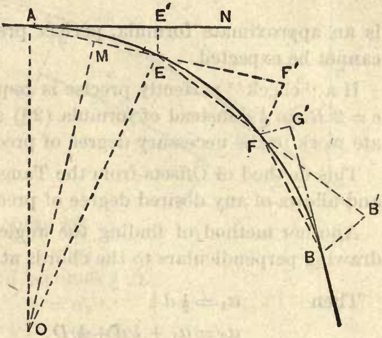
AE any chord $= c$

EE′ perp. to AE′ $= a =$ "tangent deflection"

FF′ $=$ BB′ $=$ the "chord deflection"

AO $=$ EO $= R$

Draw OM perpendicular to AE.



Then EE′ : AE $=$ ME : EO

$$a : c = \frac{c}{2} : R \qquad \text{or } a = \frac{c^2}{2R} \qquad (26)$$

FF′ $= 2a$; AF′ $=$ AE produced

When AE is a full station of 100 feet, $a_{100} = \dfrac{100^2}{2R}$ (26 A)

Field-work.

The *P.C.* and *P.T.* are assumed to have been set.

(a) Calculate a_{100}.

(b) Set point E distant 100 ft. from A and distant a_{100} from AE′(AE′ $<$ 100 ft.; AE′E $= 90°$).

(c) Produce AE to F′ (EF′ $=$ 100 ft.), and find F distant $2\,a_{100}$ from F′ (EF $=$ 100 ft.).

(d) Proceed similarly until B is reached (*P.T.*).

(e) At station preceding B (*P.T.*) lay off FG′ $= a_{100}$ (FG′B $= 90°$).

(f) G′B is tangent to the curve at B (*P.T.*).

69. Problem. *Given the degrees of two curves having the same P. C.*

Required the offset between the two curves at the end of a given chord c.

Let D_l, R_l be the D, R of flatter curve, D_s, R_s of sharper.

$$\text{For } 1^\circ \text{ curve } a_1 = \frac{c^2}{2\,R_1};\; a_l = \frac{c^2}{2\,R_l};\; a_s = \frac{c^2}{2\,R_s}$$

$$\frac{a_l}{a_1} = \frac{1}{R_l} \div \frac{1}{R_1} = \frac{D_l}{D_1} \text{ (approx.) from (3 A)}$$

$$a_l = a_1 D_l \text{ and } a_s = a_1 D_s \qquad (26\text{ B})$$

Let a_{s-l} = offset between curves $= a_s - a_l$

$$a_{s-l} = a_1 D_s - a_1 D_l$$
$$= a_1(D_s - D_l) \text{ (approx.)} \qquad (27)$$

For 1° curve and $c = 100$ $\quad a_1 = 0.873$ ft. $= \frac{7}{8}$ ft. (nearly).

$$a_{s-l} = \tfrac{7}{8}(D_s - D_l) \text{ ft. (approx.)} \qquad (27\text{ A})$$

Table XXXIII gives offsets from tangent for a 10° curve.

From this the offset for any degree is found by multiplying by $\frac{D_a}{10}$. This gives results closely approximate.

Table XXXIV is a table of corrections to be applied to results found by using Table XXXIII.

70. Problem. *Given the offset to a curve for chord of 100 ft. Required the offset for any number of chords n, each 100 ft.*

$$a = \frac{c^2}{2\,R} \quad \text{and} \quad a_{100} = \frac{100^2}{2\,R} \qquad \text{from (26 A)}$$

$$\text{for } c = 200 \qquad a_{200} = \frac{200^2}{2\,R};\; \text{for } c = n\,100 \qquad a_n = \frac{n^2\,100^2}{2\,R}$$

but n chords of 100 ft. each = chord n 100 (nearly) and

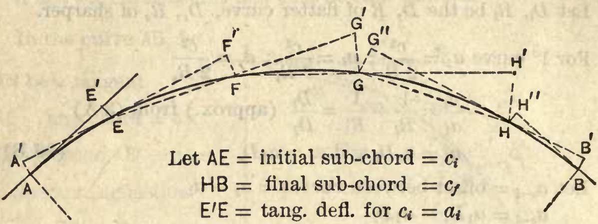
$$a_n = \frac{n^2\,100^2}{2\,R} \quad \text{or} \quad a_n = n^2 a_{100} \quad \text{(approx.)} \qquad (28)$$

This approximation may prove too rough for most field-work unless n is very small. It may be of value in plotting. It should seldom be used for other purposes.

Similarly from (27 A) $\quad a_n = \frac{7}{8}(D_s - D_l)n^2$ (roughly).

71. Problem. *Given D and the stations of P.C. and P. Required to lay out the Curve by Deflection Distances.*

When the curve begins and ends with a sub-chord.



Let AE = initial sub-chord = c_i
HB = final sub-chord = c_f
E'E = tang. defl. for c_i = a_i
H''H = " " " $c_f = a_f$

by (26) $$a_i = \frac{c_i^2}{2\,R}; \quad a_f = \frac{c_f^2}{2\,R}; \quad a_{100} = \frac{100^2}{2\,R}$$

$$\left.\begin{array}{ll} a_i : a_{100} = c_i^2 : 100^2 & a_i = a_{100}\,\dfrac{c_i^2}{100^2} \\ a_f : a_{100} = c_f^{\,2} : 100^2 & a_f = a_{100}\,\dfrac{c_f^2}{100^2} \end{array}\right\} \quad (29)$$

In general it is better to use (29) than $a_i = \dfrac{c_i^2}{2\,R}$.

72. Example. *Given P.T.* 20 + 42, *P.C.* 16 + 25, 6° curve R

Required all data necessary to lay out curve by "*Deflection Distances.*"

Calculate without Tables. Result to $\frac{1}{100}$ foot.

Radius 1° curve $= \dfrac{5730(6}{6°} \quad 955$

$$a_{100} = \frac{100^2}{2 \times 955} = 5.24$$

$2\,a_{100} = 10.47$

$a_{75} = 0.75^2 \times 5.24 = 2.95$

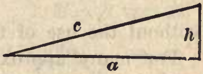
$a_{42} = 0.42^2 \times 5.24 = 0.92$

Table II. gives $a_{100} = 5.234$ (precise value)

```
1910)10000(5.235+
      955
      450
      382
      680
      573
     1070
      955
```

The distance AE′ is slightly shorter than AE. It is generally sufficient to take the point E′ by inspection simply. If desired for this or any other purpose, a simple approximate solution of right triangles is as follows:

73. Problem. *Given the hypotenuse (or base) and altitude. Required the difference between base and hypotenuse,* or in the figure, $c - a$.



$$c^2 - a^2 = h^2$$

$$(c - a)(c + a) = h^2$$

$$c - a = \frac{h^2}{c + a} = \frac{h^2}{2c} \text{ (approx.)} = \frac{h^2}{2a} \text{ (approx.)} \qquad (30)$$

Wherever h is small in comparison with a or c, the approximation is good for ordinary purposes.

Example. $c = 100$ $\qquad c - a = \frac{100}{200} = 0.50$

$h = 10$ $\qquad a = 99.50$

The precise formula gives 99.499.

74. Field-work *for § 71.*

(a) Calculate a_{100}, a_i, a_f. Remember that tangent deflections are as the *squares* of the chords.

a_{100} is found in Table II., Allen, as "tangent offset."

(b) Find the point E, distant a_i from AE′ and distant c_i from A. (AE′E = 90°.)

(c) Erect auxiliary tangent at E (lay off AA′ = a_i).

(d) From auxiliary tangent A′E produced, find point F.

(FF′ = a_{100}; EF = 100; EF′F = 90°).

(e) From chord EF produced, find point G.

(GG′ = 2 a_{100}; FG′ = FG = 100).

(f) Similarly, for each full station, use 2 a_{100}, etc.

(g) At last even station on curve, H, erect an auxiliary tangent (lay off GG″ = a_{100}; GG″H = 90°).

(h) From G″H produced, find B (B′B = a_f, etc.).

(i) Find tangent at B (HH″ = a_f; HH″B = 90°).

The values of a_{100}, a_i, a_f, should be calculated to the nearest $\frac{1}{100}$ foot.

75. Caution. The tangent deflections vary as the *squares* of the chords, not directly as the chords.

Curves may be laid out by this method without a transit by the use of plumb line or "flag" for sighting in points, and with *fair* degree of accuracy.

For calculating a_{100}, a_i, a_f, it is sufficient in most cases to use the approx. value $R_a = \frac{5730}{D_a}$. A curve may be thus laid out without the use of transit or tables.

For many approximate purposes it is well and useful to remember that the "chord deflection" for 1° curve is 1.75 ft. nearly, and for other degrees in direct proportion. A head chainman may thus put himself *nearly* in line without the aid of the transitman.

The method of "Deflection Distances" is not well adapted for common use, but will often be of value in emergencies.

76. Problem. *Given D and stations of $P.C.$ and $P.T.$ Required to lay out the curve by "Deflection Distances" when the first sub-chord is small.*

Caution. It will not be satisfactory in this case to produce the curve from this short chord. The method to be used can best be shown by example.

Let $PC = 41 + 90$.

Field-work.

Method 1.

(*a*) Set sta. 42 using $c = 10$ and $a_{10} = a_{100} \frac{10^2}{100^2}$.

(*b*) Set sta. 43 (100 ft. from 42) offsetting a_{110} from tangent.

(*c*) Set sta. 44 by chord produced and $2\,a_{100}$ offset.

Method 2.

(*a*) Set a point on curve produced backwards, using

$$c = 90 \text{ and } a_{90} = a_{100} \frac{90^2}{100^2}.$$

(*b*) Set sta. 42, using $c = 10$ and a_{10} as above.

(*c*) Set sta. 43 by chord produced and $2\,a_{100}$ offset.

A slight approximation is involved in each of these methods. Method 1 involves less labor.

77. Ordinates.

Problem. *Given D and two points on a curve.*

Required the *Middle Ordinate* from the chord joining the two points.

By (9), $M = R \text{ vers } \frac{1}{2} I$

for 100 ft. chord $M = R \text{ vers } \frac{1}{2} D$

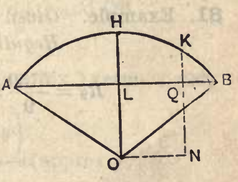
between points 2 station lengths apart $M = R \text{ vers } D.$

Let A = angle at center between any two points.

$M = R \text{ vers } \frac{1}{2} A.$

78. Problem. *Given R and c.*

Required M.



$$OL = \sqrt{R^2 - \left(\frac{c}{2}\right)^2}$$

$$HL = M = R - \sqrt{R^2 - \left(\frac{c}{2}\right)^2} \qquad (31)$$

$$M = R - \sqrt{\left(R - \frac{c}{2}\right)\left(R + \frac{c}{2}\right)} \qquad (32)$$

Table XXI., Allen, gives squares and square roots for certain numbers. If the numbers to be squared can be found in this table, use (31). Otherwise use logarithms and (32).

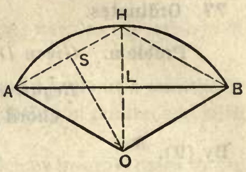
79. Problem. *Given R and C.*

Required the *Ordinate* at any *given point* Q.

Measure $LQ = q$. Then $KN = \sqrt{R^2 - q^2}$

$$LO = \sqrt{R^2 - \left(\frac{c}{2}\right)^2}$$

$$KQ = KN - LO = \sqrt{(R+q)(R-q)} - \sqrt{\left(R + \frac{c}{2}\right)\left(R - \frac{c}{2}\right)} \qquad (33)$$



80. When $C = 100$ ft. or less, an approximate formula will generally suffice.

Problem. *Given R and c.*
Required M (approx.)

$$\text{HL} : \text{AH} = \frac{\text{AH}}{2} : R$$

$$M = \frac{\text{AH}^2}{2\,R}.$$

Where AB is small compared with R,

$$\text{AH} = \frac{c}{2}\,\text{(approx.)}$$

$$M = \frac{c^2}{8\,R}\,\text{(approx.)} \tag{34}$$

81. Example. *Given $C = 100$, $D = 9°$.*
Required M.

$$R_9 = \frac{5730}{9} = 636.7$$

$$\begin{array}{r}
8 \\
\hline
5093.6)10000.(1.963 = M \\
50936 \\
\hline
490640 \\
458424 \\
\hline
322160 \\
305616 \\
\hline
16544
\end{array}$$

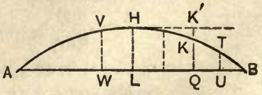
Precise value

$M = 1.965$

Table XXVII., Allen, gives middle ordinates for curving rails of certain lengths.

82. Problem. *Given R* and c.
Required Ordinate at any given point Q
Approximate Method.

I. Measure $\text{LQ} = q$



$$M = \text{HL} = \frac{\left(\frac{c}{2}\right)^2}{2\,R}\,\text{(approx.)}$$

$$\text{KK}' = \frac{\text{HK}^2}{2\,R}$$

$$\text{KK}' : M = \text{HK}^2 : \left(\frac{c}{2}\right)^2$$

Since $\mathsf{HK} = q$ (approx.) $\quad \mathsf{KK'} = \dfrac{q^2}{\left(\frac{c}{2}\right)^2} M$ (approx.) (35)

$\mathsf{KQ} = M - \mathsf{KK'}$

When $\dfrac{q}{\frac{c}{2}} = \dfrac{1}{2}$ as in figure, $\mathsf{KK'} = \dfrac{M}{4}$ and $\mathsf{KQ} = \dfrac{3}{4} M$ (approx.)

When $\dfrac{q}{\frac{c}{2}} = \dfrac{1}{4}$ $\quad \mathsf{VW} = \dfrac{15}{16} M$ (approx.)

When $\dfrac{q}{\frac{c}{2}} = \dfrac{3}{4}$ $\quad \mathsf{TU} = \dfrac{7}{16} M$ (approx.)

The curve thus found is accurately a parabola, but for short distances this practically coincides with a circle.

83. II. *Approximate Method.* Measure LQ and QB

$$M = \frac{\left(\frac{c}{2}\right)^2}{2\,R} \qquad \mathsf{KK'} = \frac{q^2}{2\,R} \text{ (approx.) from} \qquad (26)$$

$$\mathsf{KQ} = \frac{\left(\frac{c}{2}\right)^2 - q^2}{2\,R} = \frac{\left(\frac{c}{2} + q\right)\left(\frac{c}{2} - q\right)}{2\,R} \text{ (approx.)}$$

$$\mathsf{KQ} = \frac{\mathsf{AQ} \times \mathsf{QB}}{2\,R} \text{ (approx.)} \qquad (36)$$

Sometimes one, sometimes the other of these methods will be preferable.

84. Example. *Given* $C = 100$, $D = 9°$.
$M = 1.965$ from Tables.
Required, Ordinate at point 30 *ft. distant from center toward end of chord.*

I. $30 \text{ ft.} = \dfrac{30}{50} \times \dfrac{C}{2}$

$\mathsf{KK'} = \dfrac{9}{25} \times 1.965$

$$\begin{array}{r} 1.965 \\ 9 \\ \hline 25)\overline{17.685} \\ .70740 \end{array}$$

$M = 1.965$

Ordinate $= 1.258$

Precise result for data above $= 1.260$.

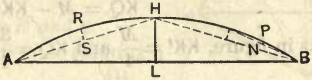
II. $\mathsf{AQ} = 80$
$\mathsf{BQ} = 20$

$R_1 = 5730.$
$R_9 = 636.7$
$2\,R_9 = 1273.4$

$$\begin{array}{r} 1273.4)1600(1.256 \\ 1273.4 \\ \hline 32660 \\ 25468 \\ \hline 71920 \\ 63670 \\ \hline 8250 \end{array}$$

85. Problem. *Given R and c.*

Required a series of points on the curve.



$$M = \mathsf{HL} = \frac{c^2}{8\,R} \text{ (approx.)}$$

$$\mathsf{RS} = \frac{\mathsf{AH}^2}{8\,R} \text{ (approx.)} \qquad \mathsf{AH} = \frac{c}{2} \text{ (approx.)}$$

$$\mathsf{RS} = \frac{\frac{c^2}{4}}{8\,R} = \frac{M}{4} \text{ (approx.)}$$

$$\mathsf{PN} = \frac{\mathsf{RS}}{4} \text{ (approx.), etc., as far as desirable.}$$

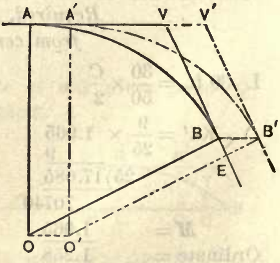
This method is useful for many general purposes, for ordinates in bending rails among others.

86. Problem. *Given a Simple Curve joining two tangents. Required* the *P.C.* of a *new curve* of the *same radius* which shall end in a *parallel tangent.*

Let AB be the given curve.

A′B′ " " required curve.

B′E $= p =$ perpendicular distance between tangents.



Join BB′.

Then $\mathsf{AA'} = \mathsf{OO'} = \mathsf{BB'}$

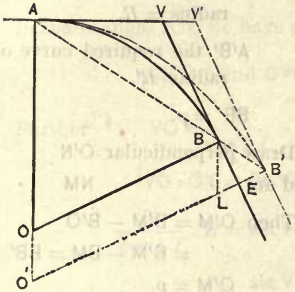
Also $\mathsf{B'BE} = \mathsf{V'VB} = I$

$\mathsf{BB'} \sin I = p$

$$\mathsf{BB'} = \mathsf{AA'} = \frac{p}{\sin I} \qquad (37)$$

When the proposed tangent is *outside* the original tangent, the distance AA is to be added to the station of the *P.C.* When *inside*, it is to be subtracted.

87. Problem. *Given a Simple Curve joining two tangents. Required* the *Radius* of a new curve which with the *same P.C.* shall end in a *parallel tangent.*



Let AB be the given curve of radius $R =$ AO.

B′E $= p =$ perpendicular distance.

AB′ the required curve, radius $= R'$.

Draw chords AB, AB′;

also line BB′;

also BL parallel to AO.

Then
$$\text{BLB}' = \text{AO}'\text{B}' = I$$
$$\text{BL} = \text{OO}'$$
$$= R' - R$$
$$= \text{B}'\text{L}$$

Therefore
$$\text{BL vers BLB}' = \text{B}'\text{E}$$
$$(R' - R) \text{ vers } I = p$$
$$(R' - R) = \frac{p}{\text{vers } I} \tag{38}$$

Since VAB = V′AB′, AB and AB′ are in the same straight line.

And with transit at A, point B′ can be set by measuring BB′ in direction AB.

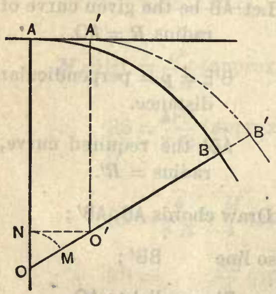
Also
$$\text{BB}' = \frac{\text{B}'\text{E}}{\sin \text{B}'\text{BE}} \quad \text{or BB}' = \frac{p}{\sin \frac{1}{2} I} \tag{39}$$

When the proposed tangent is *outside* the original tangent (as it is shown in the figure), the above formula applies, and
$$R' > R.$$

When the proposed tangent is *inside* the original tangent, the formula becomes
$$R - R' = \frac{p}{\text{vers } I} \tag{40}$$

and $R' < R$.

88. Problem. *Given a Simple Curve joining two tangents. Required the radius and P.C. of a new curve to end in a parallel tangent with the new P.T. directly opposite the old P.T.*



Let AB be the given curve of radius $= R$.

A′B′ the required curve of radius R'.

$BB' = p$.

Draw perpendicular O′N and arc NM

Then $O'M = B'M - B'O'$

$= B'M - BM = BB'$

$O'M = p$

$ON \quad \text{exsec } NOO' = O'M$

$$(R - R') \text{ exsec } I = p; \qquad R - R' = \frac{p}{\text{exsec } I} \tag{41}$$

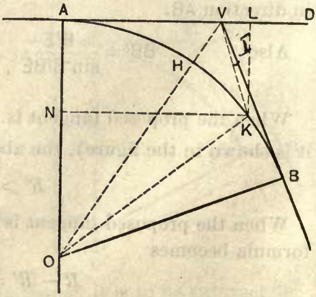
$$AA' = O'N = ON \tan NOO'$$

$$AA' = (R - R') \tan \quad I \tag{42}$$

When the new tangent is *outside* the original tangent (as in the figure), $R > R'$ and AA′ is added to the station of the *P.C.*

When the new tangent is *inside* the original tangent, $R < R'$, $R' - R = \dfrac{p}{\text{exsec } I}$, and AA′ is subtracted from station of *P.C.*

89. Problem. *To find the Simple Curve that shall join two given tangents and pass through a given point.*



With the transit at V, the given point K can often be best fixed by angle BVK and distance VK. If the point K be fixed by other measurements, these generally can readily be reduced to the angle BVK and distance VK.

90. Problem. *Given the two tangents intersecting at V, the angle I, and the point K fixed by angle* BVK $= \beta$ *and distance* VK $= b$.
Required the radius R of curve to join the two tangents and pass through K.

In the triangle VOK we have given

$$\mathrm{VK} = b \text{ and } \mathrm{OVK} = \frac{180 - I}{2} - \beta$$

Further $\mathrm{VO} = \dfrac{R}{\cos \frac{1}{2} I}$ $\quad \mathrm{OK} = R$

$$\mathrm{VO} : \mathrm{OK} = \sin \mathrm{VKO} : \sin \mathrm{OVK}$$

$$\frac{R}{\cos \frac{1}{2} I} : R = \sin \mathrm{VKO} : \cos(\tfrac{1}{2} I + \beta)$$

$$\sin \mathrm{VKO} = \frac{\cos(\frac{1}{2} I + \beta)}{\cos \frac{1}{2} I} \quad (43)$$

From data thus found, the triangle VOK may be solved for R. In solving this triangle the angle VOK is often very small. A slight error in the value of this small angle may occasion a large error in the value of R. In this case use the following **Second Method** of finding R after VOK has been found.

Find $\quad \mathrm{AOK} = \frac{1}{2} I + \mathrm{VOK} \qquad$ Also $\mathrm{DVK} = I + \beta$

Then $\quad R \text{ vers AOK} = \mathrm{LK}$

$$= b \sin \mathrm{DVK}$$

$$R = \frac{b \sin \mathrm{DVK}}{\text{vers AOK}} \quad (44)$$

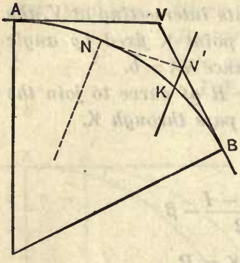
91. Problem. *Given* R, I, β (BVK).
Required b (VK).

In the triangle VOK

$$\mathrm{OK} = R; \quad \mathrm{OV} = \frac{R}{\cos \frac{1}{2} I}$$

$$\mathrm{OVK} = 90 - (\tfrac{1}{2} I + \beta)$$

Solve triangle for b.
Also find VOK and station of K if desired.



92. Problem. *To find the point where a straight line intersects a curve between stations.*

Find where the straight line V′K cuts VB at V′.

Measure KV′B.

Use V′ as an auxiliary vertex.

Find I' from V′B by (5).

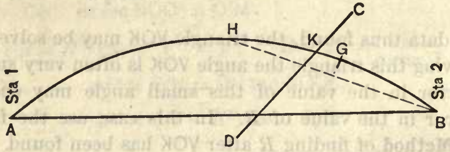
Solve by preceding problem.

93. Approximate Method.

Set the middle point H by method of ordinates.

If the arc HB is sensibly a straight line, find the intersection of HB and CD.

Otherwise set the point G by method of ordinates, and get intersection of HG and CD.

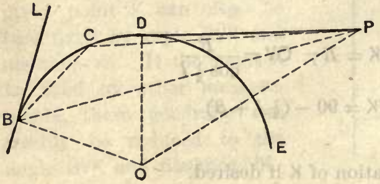


Additional points on the arc may be set if necessary, and the process continued until the required precision is secured.

The points H and G can be set without the use of a transit with sufficient accuracy for many purposes, a plumb line or flag being used in "sighting in."

94. Problem. *Given a Simple Curve and a point outside the curve.*

Required a tangent to the curve from that point.



Let BDE be the given curve.

P the point outside the curve.

BL a tangent at B.

Measure LBP, also BP.

In the triangle BPO we have given PBO, BP, BO.

Solve the triangle for BOP and OP.

Then $$\cos \text{DOP} = \frac{\text{OD}}{\text{OP}} = \frac{R}{\text{OP}}$$

$$\text{BOD} = \text{BOP} - \text{DOP}$$

From BOD find station of D from known point B.

It should be noted that if *log* OP is found, this can be used again without looking out the number for OP. Other similar cases will occur elsewhere in calculation.

When for any reason it is difficult or inconvenient to measure BP directly, the angles CBP, BCP and the distance BC may be measured and BP calculated.

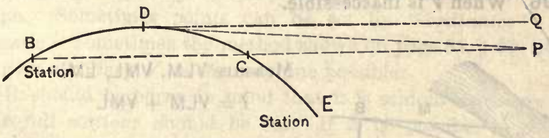
94 A. Tentative Method.

Field-work.

(*a*) From the station (B) nearest to the required point D, find by the approximate method where BP cuts the curve at C. (If E be the nearest station, produce PC to B.)

(*b*) Assume D with BD slightly greater than CD, and with transit at *P. C.* set the point D (transit point) truly on the curve.

(*c*) Move the transit to D, and lay off a tangent to the curve at D. This will very nearly strike P.

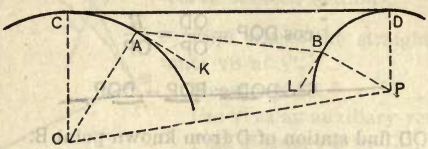


(*d*) If the tangent strikes away from P, at Q, measure QDP, and move the point D (ahead or back as the case may be) a distance *c* due to an angle at the center d = QDP. The tangent from this new point ought to strike P almost exactly.

In a large number of cases the point D will be found on the first attempt sufficiently close for the required purpose.

If a tangent between two curves is required, similar methods by approximation will be found available.

95. Problem. *Given two Simple Curves.*
Required a tangent to both Curves.



Find convenient points A and B on the given curves.

Let AK and BL be tangents.

Measure line AB and angles BAK and ABL.

Let AO $= R_l$ and BP $= R_s$ (both given).

Solve ABPO for line OP and angles AOP and BPO.

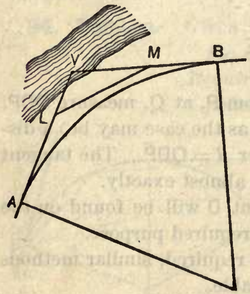
Then, $\cos \text{COP} = \dfrac{R_l - R_s}{\text{OP}}$ and $\text{DPO} = 180° - \text{COP}$

$\text{AOC} = \text{COP} - \text{AOP}$; $\text{BPD} = \text{DPO} - \text{BPO}$.

When a tangent is to connect two tracks already laid, it may be determined by a process similar to **94 A** by tentative method.

Obstacles on Curves.

96. When *V* is inaccessible.



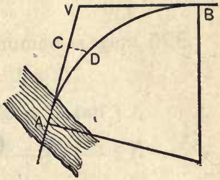
Measure VLM, VML, LM.

$I = \text{VLM} + \text{VML}$

LV and VM are readily calculated, and AL and MB determined.

In some cases the best way is to assume the position of *P.C.* and run out the curve as a trial line, and finally find the position of *P.C.* correctly by the method of formula (37).

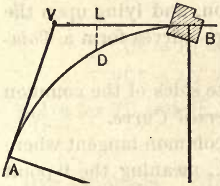
97. When the *P.C.* is inaccessible.



Establish some point D (an even station is preferable) by method of "offsets from Tangent" or otherwise.

Move transit to B (*P.T.*) and run out curve starting from D and checking on tangent VB.

98. When the *P.T.* is inaccessible.



With instrument still at *V*, set some convenient point D, move transit to *P.C.*, and run in curve to D, and then pass the obstacle at B as any obstacle on a tangent would be passed.

99. When **Obstacles on the Curve** occur so as to prevent running in the curve, no general rules can well be given. Sometimes *resetting* the transit in the curve will serve. Sometimes, if one or two points only are invisible from the transit, these can be set by "*deflection distances*," and the curve continued by "*deflection angles*," without resetting the transit. Sometimes "*offsets from the tangent*" can be used to advantage. Sometimes points can be set by "*ordinates*" from chords. Sometimes the method shown on page 54, § 92, assuming an auxiliary *V*, is the only one possible.

It should be borne in mind that it is seldom *necessary* that the *full stations* should be set. If it be possible to set any points whose stations are known and which are not too far apart, this is generally sufficient.

Finally, for passing obstacles and for solving many problems which occasionally occur, it is necessary to understand the various methods of laying out curves, and to be familiar with the mathematics of curves; and, in addition, to exercise a reasonable amount of ingenuity in the application of the knowledge possessed.

CHAPTER V.

COMPOUND CURVES.

100. When one curve follows another, the two curves having a common tangent at the point of junction, and lying upon the same side of the common tangent, the two curves form a *Compound Curve.*

When two such curves lie upon opposite sides of the common tangent, the two curves then form a *Reversed Curve.*

In a compound curve, the point at the common tangent where the two curves join, is called the *P.C.C.*, meaning the "point of compound curvature."

In a reversed curve, the point where the curves join is called the *P.R.C.*, meaning the "point of reversed curvature."

Field-work.

Laying out a compound curve or a reversed curve.

(*a*) Set up transit at *P.C.*

(*b*) Run in simple curve to *P.C.C.* or *P.R.C.*

(*c*) Move transit to *P.C.C.* or *P.R.C.*

(*d*) Set line of sight on common tangent with vernier at 0° by method of § 60.

(*e*) Run out second curve as a simple curve.

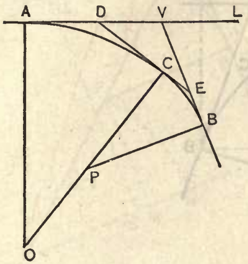
Data Used in Compound Curve Formulas.

In the curve of larger radius, OA $= R_l$; AOC $= I_l$; AV $= T_l$.

In the curve of shorter radius, PB $= R_s$; BPC $= I_s$; VB $= T_s$;

Also LVB $= I$.

101. Problem. *Given* R_l, R_s, I_l, I_s.
Required I, T_l, T_s.



Draw the common tangent DCE.

Then $I = I_l + I_s$

$$AD = CD = R_l \tan \tfrac{1}{2} I_l$$

$$EB = CE = R_s \tan \tfrac{1}{2} I_s$$

or find CD and CE, using Allen's Table III. and the correction, Table IV.

In the triangle DVE we have one side and three angles

$$DE = R_l \tan \tfrac{1}{2} I_l + R_s \tan \tfrac{1}{2} I_s \qquad (45)$$

$$VDE = I_l\,;\ VED = I_s\,;\ \text{and } DVE = 180 - I$$

Solve for VD and VE then $AV = AD + VD = T_l$
$VB = BE + VE = T_s$

102. Problem. *Given* T_s, R_s, I_s, I.
Required T_l, R_l, I_l.

$$I_l = I - I_s.$$

Find CE = EB from D_s and I_s (Tables III. and IV.)
Having given VE and all three angles
Solve for DE and DV; also find CD.

Then $T_l = AV = CD + DV$

Also $$R_l = \frac{CD}{\tan \frac{1}{2} I_l}$$

103. Problem. *Given* T_l, R_l, I_l, I.
Required T_s, R_s, I_s.

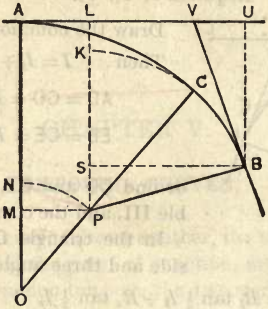
$$I_s = I - I_l.$$

Find AD = DC from D_l and I_l (Tables III. and IV.)
Having given DV and all three angles
Solve for DE and VE; also find CE.

Then $T_s = VB = VE + CE$; and $R_s = \dfrac{CE}{\tan \frac{1}{2} I_s}$

See § 105 and § 108 for other solutions of § 102 and § 103.

√ **104. Problem.** *Given* T_s, R_s, R_l, I.
Required T_l, I_l, I_s.



Draw arcs NP and KC.

Draw perpendiculars MP, LP, SB, UB.

Then

$$AM = LP$$

$$AN = R_s = KP$$

$$NM = LK = LS - KS$$

$$OP \text{ vers } NOP = VB \sin VBS - PB \text{ vers } KPB$$

$$(R_l - R_s) \text{ vers } I_l = T_s \sin I - R_s \text{ vers } I$$

$$\text{vers } I_l = \frac{T_s \sin I - R_s \text{ vers } I}{R_l - R_s} \qquad (46)$$

$$I_s = I - I_l$$

$$AV = MP + SB - UV$$

$$T_l = (R_l - R_s) \sin I_l + R_s \sin I - T_s \cos I \qquad (47)$$

√ **105. Problem.** *Given* T_s, R_s, I_s, I.
See 102
Required T_l, R_l, I_l.

$$I_l = I - I_s$$

$$R_l - R_s = \frac{T_s \sin I - R_s \text{ vers } I}{\text{vers } I_l} \qquad (48)$$

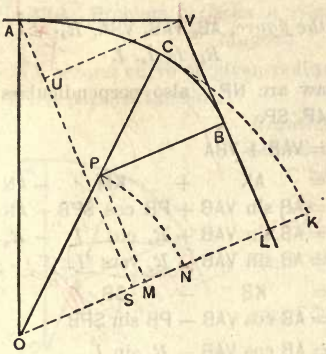
$$T_l = (R_l - R_s) \sin I_l + R_s \sin I - T_s \cos I \qquad (49)$$

√ **106. Problem.** *Given* T_l, T_s, R_s, I.
Required R_l, I_l, I_s.

46, 47

$$\tan \tfrac{1}{2} I_l = \frac{T_s \sin I - R_s \text{ vers } I}{T_l + T_s \cos I - R_s \sin I} \qquad (50)$$

$$R_l - R_s = \frac{T_l + T_s \cos I - R_s \sin I}{\sin I_l} \qquad (51)$$



107. Problem.

Given $T_l, R_l, R_s, I.$

Required $T_s, I_l, I_s.$

Draw arcs NP, KC.

Draw perpendiculars OK, AS, PM, VU.

Then LM = BP
= KN

MN = LM − LN
= KN − LN
= KL

LK = MN = KS − LS

OP vers NOP = AO vers AOK − AV sin VAS

$$(R_l - R_s) \text{ vers } I_s = R_l \text{ vers } I - T_l \sin I$$

$$\text{vers } I_s = \frac{R_l \text{ vers } I - T_l \sin I}{R_l - R_s} \qquad (52)$$

$$I_l = I - I_s$$

VB = AS − PM − AU

$$T_s = R_l \sin I - (R_l - R_s) \sin I_s - T_l \cos I \qquad (53)$$

108. Problem. *Given* $T_l, R_l, I_l, I.$

See 103

Required $T_s, R_s, I_s.$

$$I_s = I - I_l$$

$$R_l - R_s = \frac{R_l \text{ vers } I - T_l \sin I}{\text{vers } I_s} \qquad (54)$$

$$T_s = R_l \sin I - (R_l - R_s) \sin I_s - T_l \cos I \qquad (55)$$

N.T **109. Problem.** *Given* $T_l, T_s, R_l, I.$

Required $R_s, I_l, I_s.$

KS LS

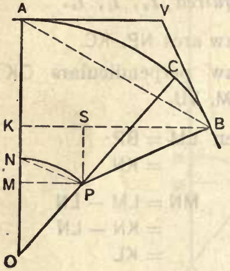
$$\tan \tfrac{1}{2} I_s = \frac{R_l \text{ vers } I - T_l \sin I}{R_l \sin I - T_l \cos I - T_s} \qquad (56)$$

AS AU VB

$$R_l - R_s = \frac{R_l \sin I - T_l \cos I - T_s}{\sin I_s} \qquad (57)$$

(55)

110. Problem. *Given, in the figure*, AB, VAB, VBA, R_s.
Required R_l, I_l, I_s, I.



Draw arc NP; also perpendiculars KB, MP, SP.

$$I = \text{VAB} + \text{VBA}$$

$$\begin{aligned}\text{NM} &= \text{AK} + \text{KM} - \text{AN}\\ &= \text{AB} \sin \text{VAB} + \text{PB} \cos \text{SPB} - \text{AN}\\ &= \text{AB} \sin \text{VAB} + R_s \cos I - R_s\\ &= \text{AB} \sin \text{VAB} - R_s \operatorname{vers} I\end{aligned}$$

1 − cos

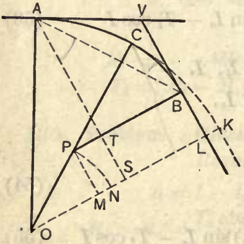
$$\begin{aligned}\text{MP} &= \text{KB} - \text{SB}\\ &= \text{AB} \cos \text{VAB} - \text{PB} \sin \text{SPB}\\ &= \text{AB} \cos \text{VAB} - R_s \sin I\end{aligned}$$

$$\tan \text{NPM} = \tan \tfrac{1}{2} I_l = \frac{\text{NM}}{\text{MP}} \qquad (58)$$

$$I_s = I - I_l$$

$$\text{OP} = R_l - R_s = \frac{\text{MP}}{\sin I_l} \qquad (59)$$

111. Problem. *Given, in the figure*, AB, VAB, VBA, R_l.
Required R_s, I_l, I_s, I.



Draw arc PN; also perpendiculars PM, AS.

$$I = \text{VAB} + \text{VBA}$$

$$\begin{aligned}\text{NM} = \text{LK} &= \text{SK} - \text{SL}\\ &= \text{OA} \operatorname{vers} \text{AOK} - \text{AB} \sin \text{VBA}\\ &= R_l \operatorname{vers} I - \text{AB} \sin \text{VBA}\end{aligned}$$

$$\begin{aligned}\text{MP} &= \text{AS} - \text{AT}\\ &= \text{OA} \sin \text{AOK} - \text{AB} \cos \text{VBA}\\ &= R_l \sin I - \text{AB} \cos \text{VBA}\end{aligned}$$

$$\tan \text{NPM} = \frac{\text{NM}}{\text{MP}} \qquad (60)$$

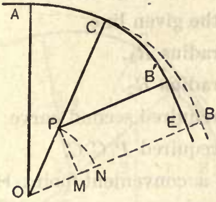
$$\tan \tfrac{1}{2} I_s = \frac{\text{NM}}{\text{MP}}$$

$$I_l = I - I_s \qquad (61)$$

$$\text{OP} = R_l - R_s = \frac{\text{MP}}{\sin I_s}$$

112. Problem. *Given a Simple Curve ending in a given tangent.*

A second curve of given radius is to leave this and end in a given parallel tangent.



Required the P.C.C.

Let AB be the given curve of radius R_l.

C be the *P.C.C.*

CB′ the second curve of radius R_s.

BE $=p=$ distance between tangents.

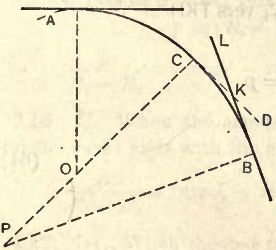
Then MN = EB $=p$.

$$\text{vers COB} = \frac{\text{MN}}{\text{OP}}$$

$$\text{vers COB} = \frac{p}{R_l - R_s} \qquad (62)$$

113. *Given, a Simple Curve of radius R_1; also a line not tangent to this curve.*

Required, the radius R_2 of a second curve to connect a given point on this curve as a P.C.C., with the given line as a tangent.



Let AC be the given curve of radius R_1.

LB the given line.

C be a point selected (as convenient or necessary) as the given *P.C.C.*

CB the required curve of radius R_2.

From C lay off auxiliary tangent CD cutting LB at K.

Measure CK and angle DKB

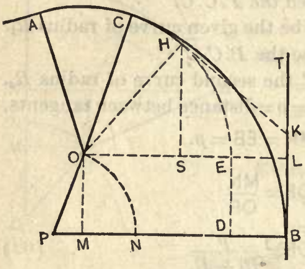
$$\text{Then } R_2 = \frac{\text{CK}}{\tan \frac{1}{2}\,\text{DKB}} \qquad (63)$$

$$\text{KB} = \text{CK}$$

This fixes the position of B, the *P.T.*, thus allowing a "check" on the field-work.

114. *Given a Simple Curve of radius R_1; also a line not tangent to this curve.*

Required, the P.C.C. of a second curve of given radius R_2 to leave this curve and join the given line as a tangent.



Let ACHE be the given curve.

TB the given line.

AO radius R_1.

PC radius R_2.

CB required second curve.

C required *P.C.C.*

From a convenient point H on the given curve lay off auxiliary tangent HK cutting TB at K.

Measure HK and angle TKH.

Then $\quad HOL = TKH$

$$LE = SL - SE$$

$$= HK \sin TKH - OH \text{ vers } HOL$$

$$p = HK \sin TKH - R_1 \text{ vers } TKH$$

Also $\quad DB = MB - MD$

$$MN = MB - NB = DB = p$$

$$MN = PO \text{ vers } OPN$$

$$\frac{p}{(R_2 - R_1)} = \text{vers } I_2 \qquad (64)$$

The angle AOH is given.

$$CPB - TKH = COH$$

$$AOH - COH = AOC = I_1$$

This serves to fix station of *P.C.C.* at C.

Also $\quad KB = KL + LB$

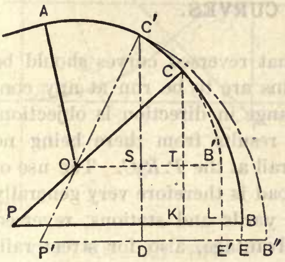
$$= HS - HK \cos TKH + LB$$

$$= OH \sin TKH - HK \cos TKH + OP \sin OPN$$

$$KB = R_1 \sin TKH - HK \cos TKH + (R_2 - R_1) \sin I_2$$

115. Problem. *Given a Compound Curve ending in a given tangent.*

Required to change the P.C.C. so as to end in a *given parallel tangent, the radii remaining unchanged.*



I. When the *new tangent* lies *outside* the *old tangent*, and the *curve ends* with curve of *larger radius.*

Let ACB be the given compound curve.

AC′B″ the required curve.

Produce C′O to P′, draw arc C′B″ and connect P′B″.

Produce arc AC to B′ and connect OB′.

Draw perpendiculars C′SD, CTK, B′LE′, and BE.

Then EB″ = E′B″ − LB

= DB″ − SB′ − (KB − TB′)

= P′C′ vers C′P′B″ − OC′ vers C′OB′

− (PC vers CPB − OC vers COB′)

$$p = (R_l - R_s) \text{ vers } I_l' - (R_l - R_s) \text{ vers } I_l$$

$$\frac{p}{R_l - R_s} = \text{vers } I_l' - \text{vers } I_l. \qquad (65)$$

116. II. When the *new tangent* lies *inside* the *old tangent*, and the *curve ends* with the curve of *larger radius.*

$$\frac{p}{R_l - R_s} = \text{vers } I_l - \text{vers } I_l'. \qquad (66)$$

117. III. When the *new tangent* lies *outside* the *old tangent*, and the *curve ends* with curve of *smaller radius.*

With a new figure it may be shown that

$$\frac{p}{R_l - R_s} = \text{vers } I_s - \text{vers } I_s' \qquad (67)$$

118. IV. When the *new tangent* lies *inside* the *old tangent*, and the *curve ends* with curve of *smaller radius.*

$$\frac{p}{R_l - R_s} = \text{vers } I_s' - \text{vers } I_s \qquad (68)$$

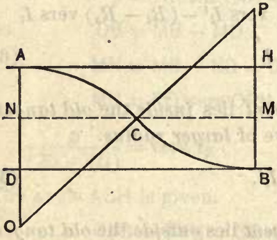
CHAPTER VI.

REVERSED CURVES.

It is considered undesirable that reversed curves should be used on main lines, or where trains are to be run at any considerable speed. The marked change in direction is objectionable, and an especial difficulty results from there being no opportunity to elevate the outer rail at the *P.R.C.* The use of reversed curves on lines of railroad is therefore very generally condemned by engineers. For yards and stations, reversed curves may often be used to advantage, also for street railways, and perhaps for other purposes.

119. Problem. *Given the perpendicular distance between parallel tangents, and the common radius of the reversed curve.*

Required the central angle of each curve.



Let AH and BD be the parallel tangents.

ACB the reversed curve.

HB $= p =$ perpendicular distance between tangents.

Draw perpendicular NM.

Let AOC $=$ BPC $= I_r$.

Then $$\text{vers AOC} = \frac{\text{AN}}{\text{AO}} = \frac{\text{BM}}{\text{PB}} = \frac{\frac{1}{2}\,\text{HB}}{\text{AO}}$$

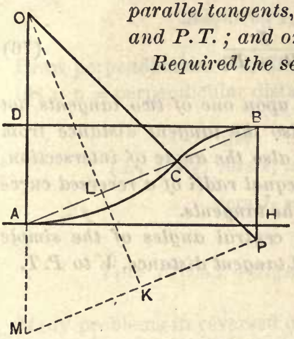
$$\text{vers } I_r = \frac{\frac{1}{2}p}{R} \qquad (69)$$

120. Problem. *Given p, I_r.*

Required R.

$$R = \frac{\frac{1}{2}p}{\text{vers } I_r} \qquad (70)$$

121. Problem. *Given the perpendicular distance p between parallel tangents, the chord distance d between P.C. and P.T.; and one radius R_1 of a reversed curve. Required the second radius R_2.*



Let ACB = reversed curve.
AH, DB parallel tangents.

$AB = d \qquad BH = p$

$OA = R_1$ and $PB = R_2$

Connect AC and CB.

AOC = BPC and ACO = PCB

ACB is a straight line.

Draw MP parallel to AB, OK perpendicular to MP.

MP = AB and AM = BP

OM : MK = AB : BH

$$R_1 + R_2 : \tfrac{1}{2} d = d : p \qquad \text{or } R_1 + R_2 = \frac{d^2}{2p} \qquad (71)$$

$$\text{When} \quad R_1 = R_2 = R \qquad R = \frac{d^2}{4p} \qquad (72)$$

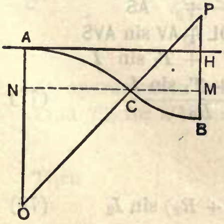
122. Problem. *Given R and p. Required d.*

$$\text{From (71)} \qquad d = \sqrt{2(R_1 + R_2)p} \qquad (73)$$

$$\text{When} \quad R_1 = R_2 = R \qquad d = \sqrt{4\,Rp} = 2\sqrt{Rp} \qquad (74)$$

123. Problem. *Given the perpendicular distance between two parallel tangents, and the central angle and radius of first curve of reversed curve.*

Required the radius of second curve.



Let ACB = reversed curve

$HB = p$; $AO = R_1$; $PB = R_2$

$AOC = CPB = I_r$

Draw perpendicular NCM.

$HB = AN + MB$

$= AO \text{ vers } AOC + BP \text{ vers } BPC$

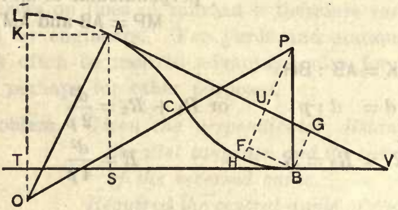
$p = R_1 \text{ vers } I_r + R_2 \text{ vers } I_r$

$$R_1 + R_2 = \frac{p}{\text{vers } I_r} \qquad (75)$$

124. Problem. *Given R_1, R_2, p.*
Required I_r.

from (75) $$\text{vers } I_r = \frac{p}{R_1 + R_2} \tag{76}$$

125. Problem. *Given a P.C. upon one of two tangents not parallel, also the tangent distance from P.C. to V, also the angle of intersection, also the unequal radii of a reversed curve to connect the tangents.*
Required the central angles of the simple curves, and tangent distance, V to P.T.



Let AV $= T_1 =$ given tangent distance
ACB $=$ required curve
AOC $= I_1$ } required angles
BPC $= I_2$ }
BV $= T_2 =$ required tangent distance

A $=$ given *P.C.*
V $=$ vertex
AVT $= I$
AO $= R_1$ } given radii
PB $= R_2$ }
VT $=$ second tangent

Draw arc AL, also perpendiculars OL, AS, AK.

Then LT $= p =$ perpendicular distance between parallel tangents and by (75) $p = (R_1 + R_2)$ vers LOC

$$\text{LT} = \text{LK} + \text{AS}$$
$$(R_1 + R_2) \text{ vers LOC} = \text{AO vers AOL} + \text{AV sin AVS}$$
$$(R_1 + R_2) \text{ vers } I_2 = R_1 \text{ vers } I + T_1 \sin I$$
$$\text{vers } I_2 = \frac{R_1 \text{ vers } I + T_1 \sin I}{R_1 + R_2} \tag{77}$$
$$I_1 = I_2 - I$$

$$\text{BV} = \text{VS} + \text{AK} - \text{TB}$$
$$T_2 = T_1 \cos I + R_1 \sin I - (R_1 + R_2) \sin I_2 \tag{78}$$

126. Problem. *Given* BV *instead of* AV, *and other data as above.*
Required I_1, I_2, *etc.*

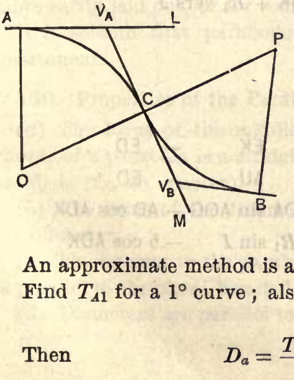
Draw perpendiculars PH, BF, BG.
UH $= p =$ perpendicular distance between parallel tangents.

$$\text{UH} = \text{FH} + \text{GB}$$
$$(R_1 + R_2) \text{ vers } I_1 = R_2 \text{ vers } I + T_2 \sin I$$
$$\text{vers } I_1 = \frac{R_2 \text{ vers } I + T_2 \sin I}{R_1 + R_2} \qquad (79)$$

$$T_1 = T_2 \cos I + R_2 \sin I + (R_1 + R_2) \sin I_1 \qquad (80)$$

Many problems in reversed curves can be simply and quickly solved by using the available data in a way to bring the problem into a shape where it becomes a case of parallel tangents with p known, and which can be solved by (75).
This is true particularly of sidings and yard problems.

127. Problem. *Given the length of the common tangent and the angles of intersection with the separated tangents.*
Required the common radius of a reversed curve to join the two separated tangents.

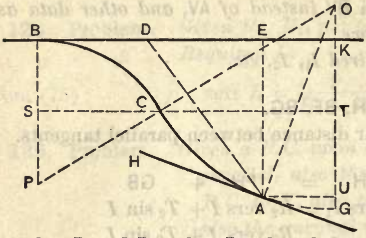


Let $\text{V}_\text{A}\text{V}_\text{B}$ = common tangent
AV_A, BV_B = separated tangents
ACB = required curve
$\text{LV}_\text{A}\text{C} = I_A$; $\text{MV}_\text{B}\text{B} = I_B$
$\text{V}_\text{A}\text{V}_\text{B} = l$

$$\text{V}_\text{A}\text{V}_\text{B} = \text{V}_\text{A}\text{C} + \text{V}_\text{B}\text{C}$$
$$l = R \tan \tfrac{1}{2} I_A + R \tan \tfrac{1}{2} I_B$$
$$R = \frac{l}{\tan \frac{1}{2} I_A + \tan \frac{1}{2} I_B} \qquad (81)$$

An approximate method is as follows : —
Find T_{A1} for a 1° curve ; also T_{B1} (Table III.)

Then $$D_a = \frac{T_{A1} + T_{B1}}{\text{V}_\text{A}\text{V}_\text{B}} \qquad \text{(approx.)}$$



128. *Given, for a reversed curve, the P.C. lying on a given tangent; also the position of another tangent not parallel; also the unequal radii, R_1 and R_2 of the reversed curve. Required the central angles I_1 and I_2; also I; also the position of P.T.*

Let ACB be the reversed curve

AH, BK, the given tangents

A, the given *P.C.*

$AOC = I_1$

$CPB = I_2$

Measure from A to some convenient point D on BK; let $AD = b$.
Measure also HAD and ADK.
Then the angle between tangents = ADK − HAD = I.
Extend arc CA to G where curve is parallel to BK.
Draw perpendiculars AE, OG, AU, SCT.

Then $KG = AE + UG$

$= AD \sin ADK + OA \text{ vers } AOG$

$p = b \sin ADK + R_1 \text{ vers } I$

From (76) $\dfrac{p}{R_1 + R_2} = \text{vers } I_2$

$I_1 = I_2 - I$

Also $BD = BK - EK - ED$

$= ST - AU - ED$

$= OP \sin CPB - OA \sin AOG - AD \cos ADK$

$= (R_1 + R_2) \sin I_2 - R_1 \sin I - b \cos ADK$

CHAPTER VII.

PARABOLIC CURVES.

129. Instead of circular arcs to join two tangents, parabolic arcs have been proposed and used, in order to do away with the sudden changes in direction which occur where a circular curve leaves or joins a tangent. Parabolic curves have, however, failed to meet with favor for railroad curves for several reasons.

1. Parabolic curves are less readily laid out by instrument than are circular curves.

2. It is not easy to compute at any given point the radius of curvature for a parabolic curve; it may be necessary to do this either for curving rails or for determining the proper elevation for the outer rail.

3. The use of the "Spiral," or other "Easement," or "Transition" curves secures the desired result in a more satisfactory way.

There are however many cases (in Landscape Gardening or elsewhere) where a parabolic curve may be useful either because it is more graceful or because, without instrument, it is more easily laid out, or for some other reason.

It is seldom that parabolic curves would be laid out by instrument.

130. Properties of the Parabola.

(*a*) The locus of the middle points of a system of parallel chords of a parabola is a straight line parallel to the axis of the parabola (*i.e.* a diameter).

(*b*) The locus of the intersection of pairs of tangents is in the diameter.

(*c*) The tangent to the parabola at the vertex of the diameter is parallel to the chord bisected by this diameter.

(*d*) Diameters are parallel to the axis.

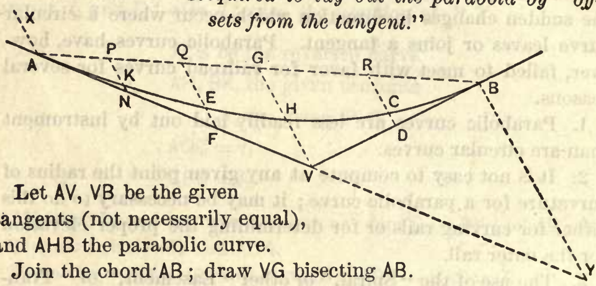
(*e*) The equation of the parabola, the coördinates measured upon the diameter and the tangent at the end of the diameter is

$$y'^2 = \frac{4\,p}{\sin^2\theta}x'$$

or

$$y^2 = 4\,p'x \tag{82}$$

131. Problem. *Given two tangents to a parabola, also the position of P.C. and P.T.*
Required to lay out the parabola by "offsets from the tangent."



Let AV, VB be the given tangents (not necessarily equal), and AHB the parabolic curve.

Join the chord AB; draw VG bisecting AB.
Draw AX, BY, parallel to VG; produce AV to Y.
Then VG is a diameter of the parabola.
AX parallel to VG is also a diameter.
The equation of the parabola referred to AX and AY as axes is

$$y^2 = 4\,p'x.$$

Instead of solving this equation engineers commonly use the proportion

$$y_1^2 : y_2^2 = x_1 : x_2 \tag{83}$$

Hence

$$\mathrm{AV}^2 : \mathrm{AY}^2 = \mathrm{HV} : \mathrm{BY}$$
$$\mathrm{AV}^2 : (2\,\mathrm{AV})^2 = \mathrm{HV} : 2\,\mathrm{GV}$$
$$1 : 4 = \mathrm{HV} : 2\,\mathrm{GV}$$

$$\mathrm{HV} = \frac{\mathrm{GV}}{2} \tag{84}$$

Next bisect VB at D.
Draw CD parallel to AX.
Then

$$\mathrm{BD}^2 : \mathrm{BV}^2 = \mathrm{CD} : \mathrm{HV}$$

$$\mathrm{CD} = \frac{\mathrm{HV}}{4}$$

Similarly, make $AN = NF = FV$

Then $$KN = \frac{HV}{9}$$

$$EF = \frac{4}{9} HV$$

In a similar way, as many points as are needed may be found.

132. Field-work.

(*a*) Find G bisecting AB.

(*b*) Find H bisecting GV.

(*c*) Find points P, Q, and N, F, dividing AG, AV, proportionately; also R and D, dividing GB and BV proportionately. Use simple ratios when possible (as $\frac{1}{2}$, $\frac{1}{3}$, etc.).

(*d*) Lay off on PN, the calculated distance KN

on QF lay off EF

on RD lay off CD

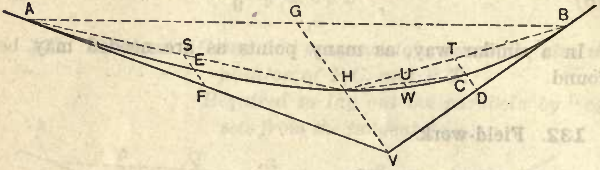
In figure opposite, $$KN = \frac{HV}{9}$$

$$CD = \frac{HV}{4}$$

$$EF = \frac{4}{9} HV$$

For many purposes, or in many cases, it will give results sufficiently close, to proceed without establishing P, Q, R; the directions of NK, EF, CD, being given approximately by eye. When the angle AVG is small (as in the figure), it will generally be necessary to find P, Q, R, and fix the directions in which to measure NK, EF, CD. When the angle AVG is large (greater than 60°) and the distances NK, EF, CD are not large, it will often be unnecessary to do this. No fixed rule can be given as to when approximate methods shall be used. Experience educates the judgment so that each case is settled upon its merits.

133. Problem. *Given two tangents to a parabola, also the positions of P. C. and P. T.*
Required to lay out the parabola by "middle ordinates."



The ordinates are taken from the middle of the chord, and parallel to GV in all cases.

Field-work.

(*a*) Establish H as in last problem.

(*b*) Lay off SE $= \frac{1}{4}$ HV; also TC $= \frac{1}{4}$ HV.

(*c*) Lay off UW $= \frac{1}{4}$ TC, and continue thus until a sufficient number of points is obtained.

The length of curve can be conveniently found only by measurement on the ground.

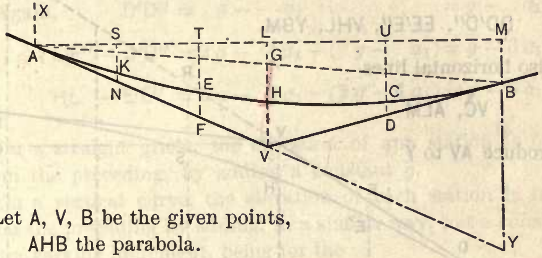
Note the difference in method between § 85 and § 133.

134. Vertical Curves.

It is convenient and customary to fix the grade line upon the profile as a succession of straight lines; also to mark the elevation above datum plane of each point where a change of grade occurs; also to mark the rates of grade in feet per station of 100 feet. At each change of grade a vertical angle is formed. To avoid a sudden change of direction it is customary to introduce a vertical curve at every such point where the angle is large enough to warrant it. The curve commonly used for this purpose is the parabola. A circle and a parabola would substantially coincide where used for vertical curves. The parabola effects the transition rather better theoretically than the circle, but its selection for the purpose is due principally to its greater simplicity of application. It is generally laid to extend an equal number of stations on each side of the vertex.

135. Problem. *Given the elevations above datum plane of grade line at the vertex, and at given points at equal distances each side of vertex, as P.C. and P.T.*

Required elevation of the vertical curve opposite the vertex; also at intermediate points.



Let A, V, B be the given points,
AHB the parabola.

Join AB; produce AV to Y.

Draw vertical lines AX, LGHV, MBY, and horizontal line ALM.

In the case of a vertical curve, the horizontal projections of AV and VB are equal, and AL = ML.

Therefore AG = GB, and AV = VY

VG is a diameter of the parabola.

AX is also a diameter.

$$\mathrm{HV} = \frac{\mathrm{VG}}{2}$$

$$\text{Elev. H} = \frac{1}{2}\left(\frac{\text{Elev. A} + \text{Elev. B}}{2} + \text{Elev. V}\right) \qquad (85)$$

The elevations of A, B, V, H, are all above "datum plane."

For intermediate points following § 131,

Let LU = UM.

$$\text{Elev. C} = \text{Elev. D} + \frac{\mathrm{HV}}{4}$$

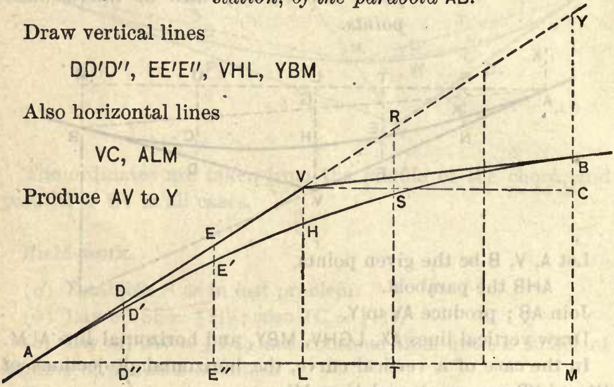
Let AS = ST = TL.

$$\text{Elev. K} = \text{Elev. N} + \frac{\mathrm{HV}}{9}$$

$$\text{Elev. E} = \text{Elev. F} + \frac{4}{9}\,\mathrm{HV}$$

136. Problem. *Given the rates of grade g of* AV ; *g′ of* VB ; *the number of stations n, half on each side of vertex, covered by the vertical curve ; also the elevation of the point* A.

Required the elevation, at each station, of the parabola AB.



Draw vertical lines

DD′D″, EE′E″, VHL, YBM

Also horizontal lines

VC, ALM

Produce AV to Y

Let a_1 = offset DD′ at the first station from A.

a_2 = " EE′ " second " " A, etc.

Then $a_2 = 2^2 a_1 = 4\, a_1$

$a_3 = 3^2 a_1 = 9 a_1$

$a_n = n^2 a_1 = \text{YB}$

$\text{YB} = \text{YC} - \text{BC}$

$$n^2 a_1 = \frac{ng}{2} - \frac{ng'}{2}$$

$$a_1 = \frac{g - g'}{2\, n} \qquad (86)$$

Due regard must be given to the signs of both g and g' in these formulas, whether + or −.

From the elevation at A we may now find the required elevations, since we have given g

and we also have a_1

$a_2 = 4\, a_1$

$a_3 = 9\, a_1$ etc.

A method better and more convenient for use is given below.

$$\mathrm{DD''} = g; \quad \mathrm{D'D''} = g - a_1$$

$$\mathrm{EE''} = 2g; \quad \mathrm{E'E''} = 2g - a_2 = 2g - 4a_1$$

$$\mathrm{VL} = 3g; \quad \mathrm{HL} = 3g - a_3 = 3g - 9a_1$$

Again, $\mathrm{D'D''} = g - a_1 \qquad = g - a_1$

$$\mathrm{E'E''} - \mathrm{D'D''} = 2g - 4a_1 - (g - a_1) = g - 3a_1$$

$$\mathrm{HL} - \mathrm{E'E''} = 3g - 9a_1 - (2g - 4a_1) = g - 5a_1$$

On a straight grade, the elevation of any station is found from the preceding, by adding a constant g.

On a vertical curve, the elevation of each station is found from the preceding by adding, in a similar way, not a constant, but a varying increment, being for the

$$\left.\begin{array}{lll} \text{1st station from} & & \mathrm{A} = g - a_1 \\ \text{2d} \quad \text{"} & \text{"} & \mathrm{A} = g - 3a_1 \\ \text{3d} \quad \text{"} & \text{"} & \mathrm{A} = g - 5a_1 \end{array}\right\} \begin{array}{l} \text{changing by successive} \\ \text{differences of } 2a_1 \text{ in} \\ \text{each case.} \end{array}$$

137. The Am. Ry. Eng. Assn. states as to length of vertical curves that " on Class *A* roads " (roads with large traffic) " rates of change of 0.10 per station on summits, and 0.05 per station in sags should not be exceeded. On minor roads 0.20 per station on summits, and 0.10 per station in sags may be used." With very steep grades, however, even higher rates than recommended by the Association may sometimes seem necessary.

The " rate of change per station " corresponds to $2a_1$ in the foregoing formulas.

Let r = rate of change per station.

Then from (86) $\qquad r = \dfrac{g - g'}{n}$

Also $$n = \frac{g - g'}{r} \qquad (87)$$

From practical considerations the vertical curve will, in general, extend an equal number of full stations on each side of the vertex.

Then $\qquad$ *n must be an even number* (not odd)

$$n \geqq \frac{g - g'}{r} \tag{88}$$

The rates of grade around the curve will be

$$g - \tfrac{1}{2}\,r; \quad g - 1\tfrac{1}{2}\,r; \quad g - 2\tfrac{1}{2}\,r, \text{ etc.}$$

Each rate differing by r from the preceding.

138. Example.

Given. Grades as follows:

Sta.	Elev.	Rate
5	117.00	
		+ 1.00
10	122.00	
		+ 0.46
15	124.30	

Assume $\quad r \leqq 0.10$

Then $\quad n \geqq \dfrac{g - g'}{0.10}$

$$\geqq \frac{1.00 - 0.46}{0.10}$$

$$\geqq 5.4$$

Use $\quad n = 6$

Then $\quad r = \dfrac{g - g'}{n}$

$$= \frac{0.54}{6} = 0.09$$

$$a_1 = \tfrac{1}{2}\,r = 0.045$$

Sta.	Elev.		
5	117.00		
	+ 1.00		$= g$
6	118.00		
	+ 1.00	1.00	$= g$
7	119.00	− 0.045	$= \frac{1}{2}\,r$
	+ 0.955	0.955	$= g - \frac{1}{2}\,r$
8	119.955	− 0.090	$-\ r$
	+ 0.865	0.865	$g - 1\frac{1}{2}\,r$
9	120.820	− 0.090	$-\ r$
	+ 0.775	0.775	$g - 2\frac{1}{2}\,r$
10	121.595	− 0.090	
	+ 0.685	0.685	
11	122.280	− 0.090	
	+ 0.595	0.595	
12	122.875	− 0.090	
	+ 0.505	0.505	
13	123.38	End of curve	
	+ 0.46		$= g'$
14	123.84		
	+ 0.46		
15	124.30		

The elevation for Sta. 15 thus obtained agrees with the elevation shown in the data. All the intermediate elevations are therefore "checked."

CHAPTER VIII.

TURNOUTS.

139. A Turnout is a track leading from a main or other track.

Turnouts may be for several purposes.

I.	*Branch Track*	(for line used as a Branch Road for general traffic).
II.	*Siding*	(for passing trains at stations, storing cars, loading or unloading, and various purposes).
III.	*Spur Track*	(for purposes other than general traffic, as to a quarry or warehouse).
IV.	*Cross Over*	(for passing from one track to another, generally parallel).

The essential parts of a turnout are

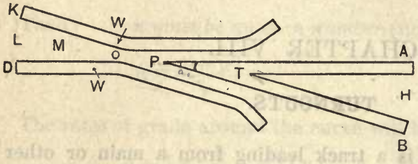
1. *The Switch.* 2. *The Frog.* 3. *The Guard Rail.*

1. Some device is necessary to cause a train to turn from the main track; this is called the "*Switch.*"

2. Again, it is necessary that one rail of the turnout track should cross one rail of the main track; and some device is necessary to allow the flange of the wheel to pass this crossing; this device is called a "*Frog.*"

3. Finally, if the flange of the wheel were allowed to bear against the point of the frog, there is danger that the wheel might accidentally be turned to the wrong side of the frog point. Therefore a *Guard Rail* is set opposite to the frog, and this prevents the flange from bearing against the frog point.

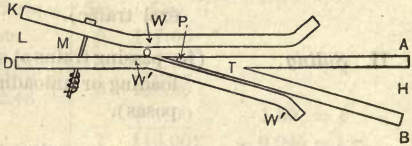
Frogs are of various forms and makes, but are mostly of this general shape, and the parts are named as follows: —



P = point
T = tongue
L = toe
H = heel
M = mouth
O = throat
WW = wings

This shows the "stiff" frog.

The "spring" frog is often used where the traffic on the main line is large, and on the turnout small. In the spring frog W′W′ is movable. AD represents the main line, and W′W′ is pushed aside by the wheels of a train passing over the turnout.



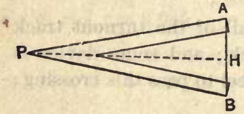
Frogs are classified by a series of standard "numbers."

The Am. Ry. Eng. Ass'n fixes the "*number*," n, by dividing length of tongue by width of heel; $n = \frac{PH}{AB} = \frac{LH}{KD + AB}$.

This is standard practice, but not adopted by all railroads.

The "*frog angle*" is the angle between the sides of the tongue of the frog = APB.

140. Problem. *Given n. Required Frog Angle F.*



$$\cot \tfrac{1}{2} F = \frac{PH}{\tfrac{1}{2} AB}$$

$$\cot \tfrac{1}{2} F = 2n \qquad (89)$$

The frog is not brought to a fine "theoretical" point or edge; but is left blunt at the "actual" point; present practice leaves the frog one half inch thick at the actual point.

Let $\quad b =$ thickness at actual point.

Then $\quad nb =$ distance, theoretical to actual point of frog.

141. The form of switch commonly used at the present time is the "split switch." Fig. A shows the switch set for the turnout, and Fig. B for the main line. With the split switch the

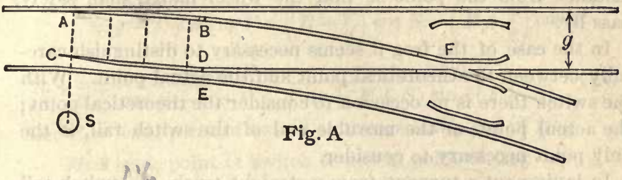
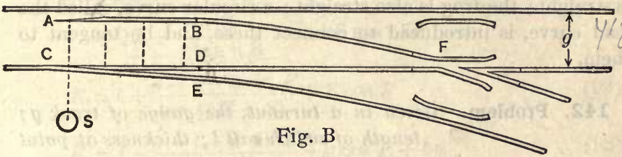


Fig. A

1½



418½

Fig. B

outer rail of the main line and the inner rail of the turnout curve are continuous. The switch rails, AB and CD, are each planed down at one end to a wedge point, so as to lie, for a portion of their length, close against the stock rail, and so guide the wheel in the direction intended. An angle, called the switch angle, is thus formed between the gauge lines of the stock rail and the switch rail, as DCE of Fig. B. The switch rails are connected by several tie rods, and one of the rods, near the point, is connected with another rod which goes to the switch stand S (or to a connection with the interlocking tower) from which the point of switch is thrown either for main track or for turnout as desired. The joint between the fixed end of the switch rail and the connecting rail, at B or D, is not bolted tight enough to prevent the slight motion of the switch rail necessary. The switch rail thus fastened at the end B is not spiked at all for its entire length, and acts as a hinged piece. Both rails thus move together, and through their entire length slide on flat steel plates provided for that purpose. The fixed (or hinged) end of this rail B is placed far enough from the stock rail to allow satisfactory spiking. This is 6½ inches, with the length of switch rail varying from 11 feet to 33 feet, in the standards of the Am. Ry. Eng. Ass'n. Gauge of track is distance from inside of rail to inside of rail. Standard gauge is 4′ 8½″.

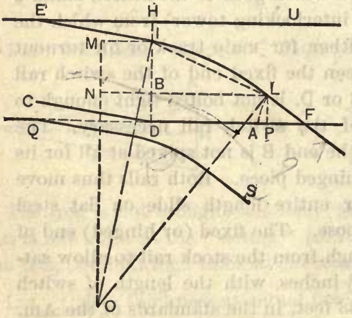
The switch rail is not planed to a fine edge but is left with appreciable thickness, frequently one quarter of an inch. The point is not left really blunt but is shaped down through a short distance from the point so that the wheel flange shall safely pass by.

In the case of the frog it seems necessary to distinguish carefully between the theoretical point and the actual point. With the switch there is no occasion to consider the theoretical point; the actual point, or the movable end of the switch rail, is the only point necessary to consider.

In laying out a turnout from a straight track, the switch rail is straight; the frog is also straight; a circular curve, called the lead curve, is introduced to connect these, and lie tangent to them.

142. Problem. *Given in a turnout, the gauge of track g; length of switch rail l; thickness at point w; heel distance between gauge sides of rails t; distance from theoretical point to toe of frog k; frog angle F and number n; thickness of frog at its point b.*

Required, radius of lead curve R; also lead E from point of switch to theoretical point of frog, and also to actual point of frog.



Let EILF and CDF be the rails of turnout,

EI and CD the switch rails.

ID is perpendicular to QDF.

Draw parallels and perpendiculars IM, LN, OM, LP, also arc LA.

Let S = switch angle HEI,

t = heel distance HI,

l = EI = QD = CD,

w = thickness of switch rail at E.

$$\sin S = \frac{t - w}{l}. \tag{90}$$

$$\text{MN} = \text{HD} - \text{HI} - \text{LP} = \quad \text{MO} \quad - \quad \text{NO}$$

$$= g - t - k \sin F = \left(R + \frac{g}{2}\right) \cos S - \left(R + \frac{g}{2}\right) \cos F$$

$$R + \frac{g}{2} = \frac{g - t - k \sin F}{\cos S - \cos F} = \frac{g - t - k \sin F}{2 \sin \frac{1}{2}(F + S) \sin \frac{1}{2}(F - S)} \tag{91}$$

Let E_t = lead, point of switch to theoretical point of frog

E_a = lead, point of switch to actual point of frog

$$\text{QF} = \text{QD} + \quad \text{BL} \quad + \quad \text{PF}$$

$$= \text{QD} + \frac{\text{IB}}{\tan \text{ILB}} + \text{LF} \cos \text{LFP}$$

$$E_t = l + \frac{g - t - k \sin F}{\tan \frac{1}{2}(F + S)} + k \cos F \tag{92}$$

$$E_a = l + \frac{g - t - k \sin F}{\tan \frac{1}{2}(F + S)} + k \cos F + bn \tag{93}$$

143. *Given for the above turnout,* F, S, g, k, E_a
Required in the figure above, the closure DA *between heel of switch rail and toe of frog; also closure* IL *of curved rail.*

$$\text{DA} = E_a - l - k - bn \tag{94}$$

$$\text{IL} = \left(R + \frac{g}{2}\right) \text{angle } (F - S) \tag{94 A}$$

Since DA as computed is independent of R and IL is dependent upon R, any lack of precision in computing R will affect the difference between DA and IL, and I will not be exactly opposite D, as assumed.

The difference between IL and DA may be conveniently found with adequate precision as follows:

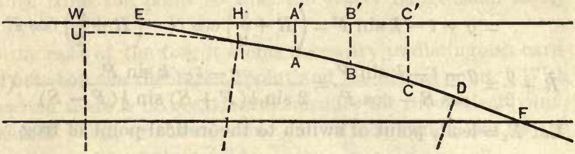
$$\text{DA} = \quad \text{NL} \quad - \quad \text{MI} \quad - \quad \text{AP}$$

$$= \left(R + \frac{g}{2}\right) \sin F - \left(R + \frac{g}{2}\right) \sin S - k \text{ vers } F$$

$$\text{IL} = \left(R + \frac{g}{2}\right) \text{angle } (F - S)$$

$$\text{IL} - \text{DA} = \left(R + \frac{g}{2}\right) [\text{angle } (F - S) - \sin F + \sin S] + k \text{ vers } F \tag{95}$$

144. *Given for a turnout, R, l, t, S, F.*
Required co-ordinates to curved rail at quarter points A, B, C.



Consider center of curve to be marked O.
Produce curve DI to U where it is parallel to EH.
Draw perpendiculars IH, AA′, BB′, CC′.

$$\text{UW} = t - \left(R + \frac{g}{2}\right) \text{ vers } S = a \qquad (96)$$

$$\text{EW} = \left(R + \frac{g}{2}\right) \sin S - l = d \qquad (97)$$

$$\text{IOD} = F - S$$

$$\text{UOA} = S + \tfrac{1}{4}(F - S) \qquad \text{UOB} = \text{UOA} + \tfrac{1}{4}(F - S)$$

$$\text{UOC} = \text{UOB} + \tfrac{1}{4}(F - S) \qquad \text{UOD} = \text{UOC} + \tfrac{1}{4}(F - S)$$

$$= F \text{ (for a check)}$$

$\text{EH} = l$ (without error of more than 0.01 foot)

$$\text{EA}' = \left(R + \frac{g}{2}\right) \sin \text{UOA} - d \qquad \text{AA}' = \left(R + \frac{g}{2}\right) \text{ vers UOA} + a$$

$$\text{EB}' = \left(R + \frac{g}{2}\right) \sin \text{UOB} - d \qquad \text{BB}' = \left(R + \frac{g}{2}\right) \text{ vers UOB} + a$$

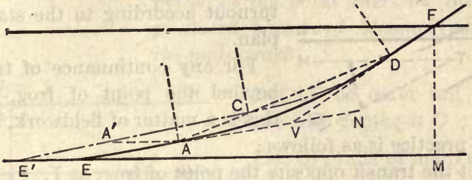
$$\text{EC}' = \left(R + \frac{g}{2}\right) \sin \text{UOC} - d \qquad \text{CC}' = \left(R + \frac{g}{2}\right) \text{ vers UOC} + a$$

145. To avoid cutting rails, one or the other of the "closure" rails between heel of switch and toe of frog may be made full feet without fractions. By lengthening the tangent of the switch rail beyond the heel, the lead is increased; by lengthening the tangent of the frog back of the toe, the lead is decreased. The leads found in this way are called "practical leads"; the leads previously considered are called "theoretical leads."

The Am. Ry. Eng. Ass'n has adopted certain combinations of switches and frogs as "standard" and calculated a table of radii, leads (both theoretical and practical), and co-ordinates of quarter points. Table XXII A and XXII B show these.

Problem. *Given the increase of lead necessary to secure practical lead; also F, S, l, t, k, g.*

Required, increase of tangent past heel of switch.



Let EM = theoretical lead; E′M = practical lead

EADF and E′A′CDF be the corresponding turnouts

EA $= l$; E′C $= l'$

Draw parallel AA′; chords AD, CD; tangents AVN, DV

Then E′E = given increase of lead; A′C = required increase of tangent; DVN $= F - S$

ADV = CDV $= \frac{1}{2}(F - S)$ and AC and AD coincide

In triangle A′AC, A′CA $= \frac{1}{2}(F - S)$; CA′A $= S$; A′A = E′E

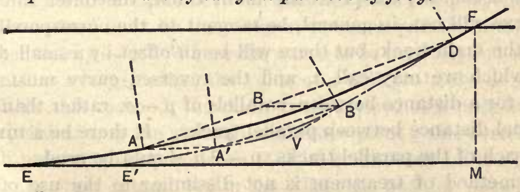
$$A'C = \frac{E'E \sin \frac{1}{2}(F + S)}{\sin \frac{1}{2}(F - S)} = l' - l \qquad (98)$$

Following (91) $$R + \frac{g}{2} = \frac{g - t - (l' - l) \sin S - k \sin F}{2 \sin \frac{1}{2}(F + S) \sin \frac{1}{2}(F - S)} \qquad (99)$$

For finding co-ordinates of quarter points, use instead of (96) the following $$a = t + (l' - l) \sin S - \left(R + \frac{g}{2}\right) \text{vers } S \qquad (96\text{ A})$$

146. Problem. *Given the decrease of lead necessary to secure practical leads; also F, S, l, t, k, g.*

Required increase of tangent past toe of frog.

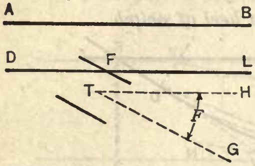


Let DF $= k$ and B′F $= k'$ From the figure it may be found that

$$B'D = \frac{E'E \sin \frac{1}{2}(F + S)}{\sin \frac{1}{2}(F - S)} = k' - k \qquad (100)$$

$$\frac{g - t - k' \sin F}{2 \sin \frac{1}{2}(F + S) \sin \frac{1}{2}(F - S)} = R + \frac{g}{2} \qquad (101)$$

147. It has become the custom to stake out the position of the frog point F. From this point F, a good track foreman will work backward and lay out the turnout according to the standard plan.



For any continuance of turnout beyond the point of frog, where this is a matter of fieldwork, a very common practice is as follows:

(*a*) Set the transit opposite the point of frog, at T.

(*b*) Lay off on the transit (on the proper side of 0°) the value of the frog angle *F*.

(*c*) Sight in the direction TH, parallel to AB.

(*d*) Turn off HTG = *F*.

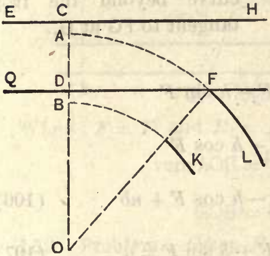
(*e*) The transit then sights along TG with vernier at 0°.

Any curve desired may then be laid off conveniently by deflection angles, and this curve will connect at T (opposite F) with whatever arrangement of track extends backward from the point of frog to the point of switch. Where the line in advance of F is new location, TG is the basis for that location; TG is either continued as a straight line, or it becomes the tangent to a desired curve and the transit is already set on TG with the vernier at 0°. When the turnout is to connect with some track parallel to the main track, the simplest method is to resolve the problem into a case of reversed curves with parallel tangents, by the following method, similar to that of § 125. If the curve used beyond F is extended backward toward the point of switch until it becomes parallel to the main track, the outer rail of this curve will not, in general, be tangent to the corresponding rail of the main track, but there will be an offset by a small distance which we may call a, and the reversed curve must be figured for a distance between parallels of $p - a$, rather than p, the actual distance between parallel tracks. If there be a turnout at each of the parallel tracks, $p - 2a$ should be used.

This method of treatment is not dissimilar to the use of p and q in spirals, and has value in many cases other than those of parallel tracks; several cases will be treated in the next chapter.

The method of finding a follows.

148. Problem. *Given a curve of radius R to be used beyond the frog; also F, n, g, b.*
Required the co-ordinates (from the frog point) of the point where the given curve produced backward becomes parallel to the main track.



Let LF be outer rail of given curve with center at O

EH, QF, rails of main track

Produce curve LF to A where it becomes parallel to EH, and draw OC perpendicular to EH

Let CA $= a$; FD $= y$

AD $= x$

To theoretical point of frog $y_t = \left(R + \frac{g}{2}\right) \sin F$ (102)

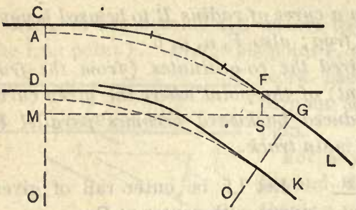
To actual point $y_a = \left(R + \frac{g}{2}\right) \sin F + nb$ (103)

$$x = \left(R + \frac{g}{2}\right) \text{vers } F \qquad (104)$$

$$g - x = a = g - \left(R + \frac{g}{2}\right) \text{vers } F \qquad (105)$$

If the curve produced backward becomes parallel above the rail ECH, the value of a becomes *minus* and where the problem is for a reversed curve between parallel lines the perpendicular distance used must be the distance between parallel lines $p + a$ rather than $p - a$. Where the curve to be used beyond the frog point has a large radius, the value of a will probably be *minus*.

With this method, the main track is used as a base-line and the point of frog the standard reference point; this corresponds with present good practice. If F be staked out on the ground or its position determined in the office, the position of point A is readily found by y, x, a. Conversely, if the position or station of A is found by computation, F is also determined. The entire split-switch turnout may then be laid out from F as a starting point and from QF or EH as reference lines.



149. If it be desired to use greater precision, and take into account the fact that the frog is straight from theoretical point F to heel G, and to make the curve beyond the frog tangent to FG at G,

Let FG $= h$

Then $\quad \mathrm{AD} = x = \left(R + \frac{g}{2}\right)$ vers $F - h \sin F$

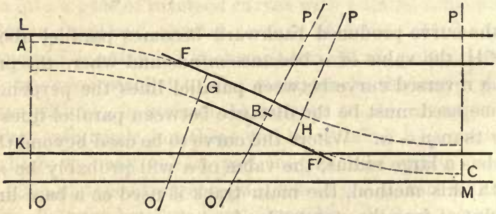
$$\mathrm{FD} = y_t = \left(R + \frac{g}{2}\right) \sin F - h \cos F$$

$$y_a = \left(R + \frac{g}{2}\right) \sin F - h \cos F + nb \qquad (106)$$

$$g - x = g - \left(R + \frac{g}{2}\right) \text{vers } F + h \sin F = a \qquad (107)$$

150. Problem. *Given the radii R_1, R_2, of two parts of a reversed curve extending from heel of frog to heel of frog between parallel tracks; also unequal frog angles F, F'; also h, h', also perpendicular distance between tracks p, and gauge g.*

Required angles GOB *and* BPH.



Let G and H be heels of frogs F and F′

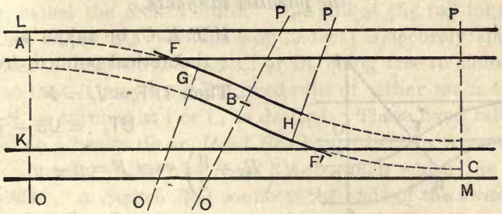
$\mathrm{LK} = p$; $\mathrm{OB} = R_1$; $\mathrm{PB} = R_2$

Find $\mathrm{LA} = a_1$ and $\mathrm{MC} = a_2$ by (107)

From (76) $\quad$ vers AOB $= \dfrac{p - a_1 - a_2}{R_1 + R_2}$ $\qquad (108)$

$$\mathrm{GOB} = \mathrm{AOB} - F \text{ and } \mathrm{BPH} = \mathrm{AOB} - F'$$

151. More commonly the two frogs will have the same number and the radii of the reversed curve will be the same.



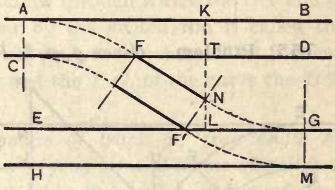
When $F = F'$ and $R_1 = R_2$

$$\text{vers AOB} = \frac{p - a - a}{R_1 + R_2} = \frac{\frac{1}{2}p - a}{R} \qquad (109)$$

$$\text{GOB} = \text{BPH} = \text{AOB} - F$$

152. **Problem.** *Given $F = F'$, n, b; also p, g.*

Required the length l, of tangent between the two frogs.



Let F and F′ be theoretical points of frogs

Draw KTNL perpendicular to AB

Then

$$\text{TN} = \text{KL} - \text{KT} - \text{NL}$$

$$\text{FN} \sin \text{TFN} = p - g - \text{F'N} \cos \text{F'NL}$$

$$l \sin F = p - g - g \cos F$$

$$l = \frac{p - g - g \cos F}{\sin F} \qquad (110)$$

l is the distance from the theoretical point at F to point N opposite the theoretical point at F′

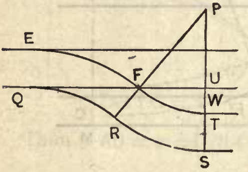
The above solution holds good whatever be the turnout used.

For a crossover between existing tracks, if the distance FF′ be calculated, both frog points can be located and the entire turnout staked out without transit.

from (30) $\quad \text{FF'} = l + \dfrac{g^2}{2\,l}$ (approx.)

The distance from actual point of one frog to the actual point of the other $= \text{FF'} - 2\,nb$.

153. Problem. *Given F, n, p, g.*
Required the radius of curve R_2, to connect the parallel tangents.



If *P.R.C.* be taken at F, the theoretical point of frog.

Then TPF $= I_r = F$

$$\text{UT} = \text{US} - \text{TS}$$

$$\left(R_2 - \frac{g}{2}\right) \text{ vers } F = p - g$$

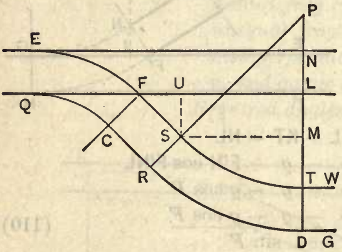
$$R_2 - \frac{g}{2} = \frac{p-g}{\text{vers } F}. \quad (111)$$

154. *Second Solution.* UT $= p - g$; PW $= R_2 - \dfrac{p}{2}$

by (118) PW = UT $2\,n^2$

$$R_2 - \frac{p}{2} = (p-g)2\,n^2. \quad (112)$$

155. Problem. *Given g, p, l, F.*



Required R_2 of curve to connect parallel tangents.

Let F be the theoretical point of frog; l the distance from theoretical point of frog to S opposite *P. C.* of curve.

Draw the perpendiculars SU, SM.

Then SU = LM = NT − NL − MT

FS sin UFS = NT − NL − PS vers SPT

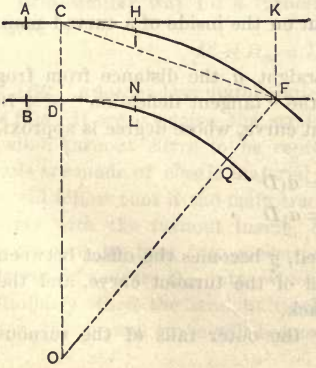
$$l \sin F = p - g - \left(R_2 - \frac{g}{2}\right) \text{vers } F$$

$$R_2 - \frac{g}{2} = \frac{p - g - l \sin F}{\text{vers } F} \quad (113)$$

By taking FS or $l = h$ (the distance from theoretical point to heel of frog) formula (113) covers the case where the reversing curve starts from the heel of frog.

156. A form of switch formerly in common use is the "*Stub-Switch*," which is formed by two rails, one on each side of the track, called the *Switch Rails.* One end of the rail for a short distance AC or BD (perhaps 5 or 10 feet) is securely spiked to the ties, the rest of the rail CI or DL being free to slide on the ties, so that it may meet the fixed rails of either main track at H or N, or turnout at I or L, as desired. These fixed rails, supported on a heavy tie or *Head Block*, are held by a casting, or piece of metal called the *Head Chair*, upon which the switch rail slides. A *Switch Rod* connects the ends of the switch rails with the *Switch Stand.* Since one end of the rail is spiked down, when the free end is drawn over by the switch rod the rail is sprung into a curve which may with slight error be considered a circular curve, tangent to the main line (if this be straight). In the stub-switch the outer rail of the turnout is assumed to be tangent both to the main track at C and to the frog at its point F. The distance through which the free end of the rail is drawn or *thrown* by the switch rod is called the *Throw* of the switch; this will be 5 or 6 inches. The free end of the rail is called the *Toe*, and the *P.C.* of the curve the *Heel* of the switch.

157. Problem. *Given gauge of track g; frog angle F; number of frog n; and throw of switch t. Required for a stub-switch, radius of turnout curve R; length of switch rail l; and* DF, *the lead E.*



Draw perpendicular FK

$$COF = F$$

$$FO = \frac{CD}{\text{vers } COF}$$

$$R + \frac{g}{2} = \frac{g}{\text{vers } F} \qquad (114)$$

$$DF = OF \sin DOF$$

$$E = \left(R + \frac{g}{2}\right) \sin F \qquad (115)$$

From (26) $t = \dfrac{l^2}{2R}$ (approx.)

$$l = \sqrt{2Rt} \qquad (116)$$

$$DF = CD \cot CFD; \quad E = g \cot \tfrac{1}{2} F; \quad E = 2\,gn \qquad (117)$$

$$DF^2 = FO^2 - DO^2$$

$$E^2 = \left(R + \frac{g}{2}\right)^2 - \left(R - \frac{g}{2}\right)^2$$

$$E^2 = \left(R + \frac{g}{2} + R - \frac{g}{2}\right)\left(R + \frac{g}{2} - R + \frac{g}{2}\right) = 2\,Rg$$

$$R = \frac{E^2}{2\,g} = \frac{(2\,gn)^2}{2\,g} = 2\,gn^2 \qquad (118)$$

$$R = En$$

$$l = \sqrt{2\,Rt} = \sqrt{4\,n^2gt} = 2\,n\sqrt{gt}$$

These formulas in § 156 and § 157 apply only in the case of the stub-switch, and are not to be used for split-switch turnouts.

158. Problem. *Given the degree D of a stub-switch turnout from a straight track.*
Required the degree of curve D' for a stub-switch turnout from a curved main track of degree D_m, F, n, g, remaining the same.

It may be shown that for a turnout to the inside of the curve

$$D' = D + D_m \text{ (approx.)} \qquad (119)$$

for a turnout outside the curve

$$D' = D - D_m \text{ (approx.)}$$

except that $\quad D' = D_m - D$ (approx.)

when $\quad D_m > D$

Take the case of the turnout on the inside of a curved main track.

When the main track is straight, g, the distance from frog point to the rail opposite, is the "tangent deflection" of § 70 for the outer rail of the turnout curve, whose degree is approximately D.

From (26 B) $\quad a = a_1 D$

so that $\quad g = a_1 D$

When the main line is curved, g becomes the offset between two curves, one the outer rail of the turnout curve, and the other the outer rail of main track.

Assuming the chords c for the outer rails of the turnout

curves to be equal in the two cases of straight main track and curved main track

by (27) $$a_{s-l} = a_1(D_s - D_l)$$

and the degree of the turnout curve must be such that

$$g = a_1(D' - D_m)$$

The values of c and E are nearly equal; so that what is true of the chord in this relation is also true of E (very closely). Therefore for a given value of E

$$D' = D + D_m \text{ (approx.)}$$

Furthermore the length of turnout curve is equal to c (very closely); for the given length $= c$ the angle $I = \frac{cD}{100} = F$, and since $D' - D_m = D$, the difference in angle $\frac{cD'}{100} - \frac{cD_m}{100} = \frac{cD}{100} = F$, so that the frog angle is not changed (materially).

Similar consideration of the two cases of turnout outside the curve of main track will show the expressions above to be true.

159. Example. Required the stub-switch turnout from a 3° main line curve using a No. 9 frog.

Table XXII shows for a No. 9 frog the

$$\begin{array}{lll} \text{degree of curve} & = & 7°\,31' = D \\ \text{The degree of main line} & = & \underline{3°\,00'} = D_m \\ \text{degree of turnout} & = & 10°\,31' = D' = D + D_m \\ \text{By precise formula} & & 10°\,32' = D' \end{array}$$

In a similar way for a turnout on the outside of the same curve

$$D' - D_m = D = 4°\,31'$$

160. Another less mathematical, but very useful illustration is this: If we conceive the straight main track and the stub-switch turnout curve to be represented by a model where the rails are made of elastic material; using a "bending process" it will follow that if the main track rails be bent into a circular curve with the turnout inside, then the rails of the turnout curve will be bent into a sharper curve, and sharper by the degree of curve D_m into which the straight track is bent. Similarly when the straight track is bent in the opposite direction, the turnout curve will become flatter by the amount of D_m.

161. Problem. *Given F, n, k, g, R_m, D_m.*
Required the split-switch turnout from the given curved main track.

Tables XXII A and XXII B give, for various numbers of frog, the length of switch rail l, heel distance t, lead E, radius R and degree D of lead curve, length of frog from toe to theoretical point k; also co-ordinates to quarter points. These tables show the standards adopted by the Am. Ry. Eng. Assn. for turnouts from tangents.

For turnouts from curved tracks, applying the "bending process," l, t, k, E remain unchanged in length; this is true also of the co-ordinates at the quarter points, the y values being measured along the curved main rail and x values normal to this rail; straight rails become curved to the degree of the curved main track, track or rails already curved are bent into curves sharper than before by D_m (or flatter by D_m depending upon which side of the main track the curved track lies).

The degree of lead curve $D' = D \pm D_m$

The frog remains straight necessarily; the distance k is small for all sharp lead curves, and the resulting error will be small. Furthermore the straight frog is laid as part of the main track which is assumed to be curved, so that a correct mathematical treatment is impracticable.

The switch-rail can be and should be curved to the degree D_m. It is better to curve it in a bending machine, but it is often laid straight and the traffic depended upon to curve it to a fit against the stock rail.

162. Example. For a number 9 frog, Table XXII A gives

$$l = 16' \, 6''; \; t = 6\tfrac{1}{4}''; \; k = 6'; \; h = 10'.$$

Table XXII B gives for "practical leads"

$$D = 9^\circ \, 29'; \; E_a = 72.28\,;$$

the co-ordinates are

28.75, 1.02 ; 40.98, 1.76 ; 53.19, 2.75

In using a number 9 turnout inside a 2° curved track

$$D' = 9^\circ \, 29' + 2^\circ = 11^\circ \, 29'$$

The other linear dimensions remain unchanged.

163. Problem. *Given the radial distance p between a given curved main track and a parallel siding, also frog angle F (or number n), and gauge of track g.*

Required the radius R_2' of second curve to connect point of frog with siding.

I. When the siding is **outside** the main track.

Let CM be the inner rail of the given main line.

CFT inner rail of turnout.

R_m = radius of main line.

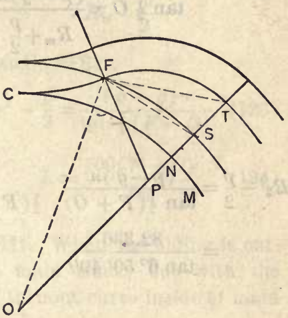
R_2' = radius of turnout.

p = TN = radial distance.

Connect FT, FO. Join FS.

Let FOT $= O$.

In triangle FTO,



$$\text{FO} = R_m + \frac{g}{2} \qquad \text{TO} = R_m - \frac{g}{2} + p$$

also
$$\text{OFT} + \text{OTF} = 180° - \text{FOT} = 180° - O$$
$$\text{OFT} - \text{OTF} = \text{OFT} - \text{PFT} = F$$

$$\tan \tfrac{1}{2}(\text{OFT} + \text{OTF}) : \tan \tfrac{1}{2}(\text{OFT} - \text{OTF}) = \text{TO} + \text{FO} : \text{TO} - \text{FO}$$
$$\cot \tfrac{1}{2} O \quad : \tan \tfrac{1}{2} F \quad = 2 R_m + p : p - g$$

$$\tan \tfrac{1}{2} O = \frac{p-g}{2 R_m + p} \cot \tfrac{1}{2} F = \frac{p-g}{R_m + \frac{p}{2}} \cdot \frac{\cot \frac{1}{2} F}{2} = \frac{(p-g)n}{R_m + \frac{p}{2}} \quad (120)$$

Similarly $\qquad \text{FPT} = F + O$

In the triangle PFS, $\tan \frac{1}{2}(F + O) = \dfrac{(p-g)n}{R_2' - \frac{p}{2}}$;

$$R_2' - \frac{p}{2} = \frac{(p-g)n}{\tan \frac{1}{2}(F+O)} \quad (121) \qquad L = \frac{100\,(F+O)}{D_2'} \quad (121\text{ A})$$

Since the main track is assumed to be curved past the frog and the frog is necessarily laid straight, it seems an unnecessary refinement to assume the frog straight from point to heel in this case.

164. Example.

Turnout from curve *outside* the main track.

Let $D_m = 4$; $n = 8$; $p = 15$; $g = 4.708$

Precise Method.

$$\tan \tfrac{1}{2} O = \frac{(p-g)n}{R_m + \frac{p}{2}} = \frac{10.292}{1440.2} \times 8 = \frac{82.336}{1440.2}$$

	82.336	log 1.915590
	1440.2	log 3.158422
	$\frac{1}{2} O = 3° 16' 19''$	tan 8.757168
$R_2' - \frac{p}{2} = \frac{(p-g)n}{\tan \frac{1}{2}(F+O)}$	$\frac{1}{2} F = 3° 34' 30''$	
	$\frac{1}{2}(F+O) = 6° 50' 49''$	tan 9.079448
$= \frac{82.336}{\tan 6° 50' 49''}$	82.336	log 1.915590
	685.7	log 2.836142
	$\frac{1}{2} p = 7.5$	
	$R_2' = 693.2$	
	$D_2' = 8° 16'.4$	

$$L = \frac{100(F+O)}{D_2'} = \frac{100 \times 13° 41' 38''}{8° 16'.4} = 165.5$$

Approximate Method.

Apply the "bending process" of p. 93.

In the case of a turnout from a straight main track, where $n = 8$ and $p = 15$,

from (112) $R_2 - \frac{p}{2} = (p-g)\, 2\, n^2$

$$= (15.0 - 4.708)\, 2 \times 64 = 1317.4$$

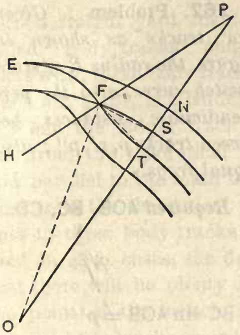
$R_2 = 1324.9$; $D_2 = 4° 19.5$; $F = 7° 09'$ (Table XXII.)

$L = \frac{100 \times 7° 09'}{4° 19.'5} = 165.3$ for straight tracks

$D_2' = D_2 + D_m$

$= 4° 19' + 4° = 8° 19'$ (8° 16' precise method)

$L = 165.3$ as with straight track (165.5 precise method).



165. II. When the siding is inside the main track.

In a similar fashion it may be shown, using this figure, that

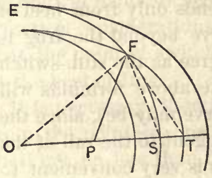
From triangle OFT

$$\tan \tfrac{1}{2} O = \frac{(p-g)n}{R_m - \frac{p}{2}} \qquad (122)$$

From triangle PFS

$$R_2' - \frac{p}{2} = \frac{(p-g)n}{\tan \frac{1}{2}(F-O)} \qquad (123)$$

$$L = \frac{100(F-O)}{D_2'} \qquad (124)$$



166. III. When the siding is outside the main track, but with the center of turnout curve inside of main track.

Let EFS be the outer rail of main track.

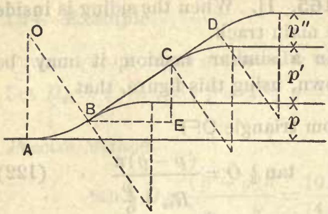
FT the inner rail of turnout.

From triangle OFT $\tan \frac{1}{2} O = \dfrac{(p-g)n}{R_m + \frac{p}{2}}$ (125)

From triangle PFS $R_2' - \dfrac{p}{2} = \dfrac{(p-g)n}{\tan \frac{1}{2}(F+O)}$ (126)

$$L = \frac{100(F+O)}{D_2'} \qquad (127)$$

With both § 165 and § 166, approximate results may be reached, by using the "bending method" of p. 93. Where the radius R_2 of the second curve is large and p is small, the approximate method will be sufficiently close; where p is large, the precise method will be necessary. Experience will determine in what cases it will be sufficient to use the approximate results, and where precise formulas should be used.



167. Problem. *Given for tracks as shown in figure, the radius R of stub-switch curve, also the perpendicular distances between tracks p, p', p''; also equal frogs.*

Required AOB, BC, CD.

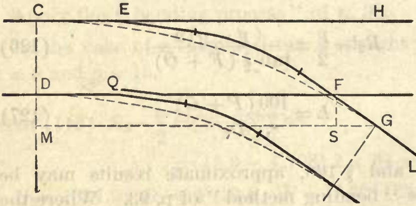
From (71) $\text{vers AOB} = \dfrac{\frac{1}{2}p}{R}$

also $\qquad \text{BC} \sin \text{CBE} = \text{CE} \qquad$ or $\text{BC} \sin \text{AOB} = p'$

$$\text{BC} = \frac{p'}{\sin \text{AOB}}; \quad \text{and CD} = \frac{p''}{\sin \text{AOB}} \qquad (128)$$

Since the standard turnout curve extends only from heel of switch to toe of frog, any convenient curve beyond the frog is appropriate. If a curve of the same degree as the stub-switch curve be used beyond the frog point, the above formulas will apply (whatever the standard turnout curve may be), since the outer curved rail extended back comes tangent to the rail of the main track. The stub-switch curve thus is very convenient to use.

If it seems advisable to consider the frog straight from point at F to heel at G in the figure below,



Let $\text{FG} = h$

$\text{CM} = g + h \sin F$

$R = 2\,n^2(g + h \sin F)$

This is the radius of the curve whose outer rail is tangent to the rail of the main track and also to the frog at its heel G.

For a series of tracks like those above when the main track is curved, the computations may be made for straight tracks and the bending process applied. Just how far this process may be carried will be determined by experience.

In a freight yard the tracks on which cars are stored are called "body tracks" and the track which leads to, and connects with, these body tracks is called the "ladder track." The track AD, § 167, is a ladder track.

When the ladder track leaves the main track in a straight line from the theoretical point of frog, if the body tracks are laid parallel to the main track, they may be laid out in straight lines from the theoretical point of the frogs used for the turnouts to these body tracks. With frogs of numbers commonly used in such cases, the distance BC or CD will be sufficient so that there will be plenty of room between the heel of frog and the point of the switch rail following it. For example, the parallel body tracks are seldom less than 12 ft. between centers; with a No. 8 frog and $p = 12$ ft. BC will be 96.4 ft.

The practical lead (Table XXII B) will be 67.6 ft.; from theoretical point to heel of frog will be (Table XXII A) 8.8 ft. Practical considerations involving bending the stock rail demand that the point of switch shall lie fully 4 ft. beyond the rail just at the heel of frog.

It is necessary, therefore, that on a ladder track the distance from the point of one switch to the point of the next switch shall not be less than $67.6 + 8.8 + 4.0 = 80.4$ ft. where a No. 8 frog is used. Since there is 96.4 ft. available and 80.4 ft. only needed, this arrangement of tracks leaves ample room.

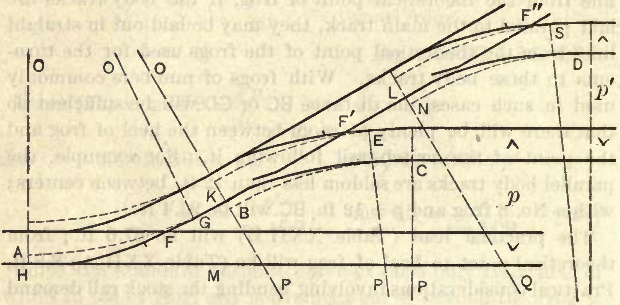
If the angle the ladder track makes with the main track be increased, the body tracks are lengthened and the ladder track becomes shorter; both of these results are of value. In the case taken above the angle can be increased until $\sin I = \frac{12}{80.8}$ or $I = 8° 32'$, or in general terms let q = clearance required from heel of frog to next point of next switch, then

$$\sin I \overline{\angle} \frac{p}{E_t + h + q}$$

It will be necessary also that I shall not exceed the value of AOB in § 167.

The arrangement of a series of body tracks and the ladder tracks allows an opportunity, in many cases, for careful study and much ingenuity; an extended treatment here will not be justified.

168. Problem. *Given for tracks shown in figure the radius R of the curve beyond the heel of frog; also p, p' between parallel tracks; also F, n, g. Required angle AOK and distance F′F″.*



Let GK with its center at O be outer rail of the given curve of radius R.

Produce this curve to A when it is parallel to HM.

Let BC with center at P, and ND with center at Q, be similar curves produced.

Let FG, F′E, F″S be straight lines from theoretical point to heel of frogs.

$$OA = R + \frac{g}{2}; \; BP = NQ = R - \frac{g}{2}; \; AH = KB = LN = a.$$

Find a by (107).

$$OP = R + \frac{g}{2} + a + R - \frac{g}{2} = 2R + a$$

Then by (76) $$\text{vers AOK} = \frac{p}{2R + a} \qquad (129)$$

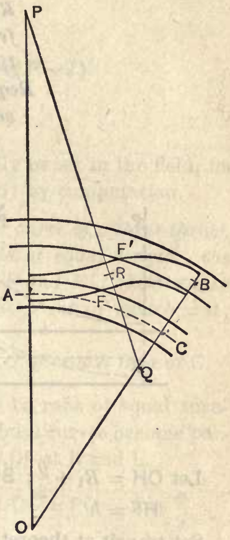
by (128) $$KL = \frac{p'}{\sin \text{AOK}}$$

Since KF′ = LF″ $$KL = F'F'' = \frac{p'}{\sin \text{AOK}} \qquad (130)$$

169. Problem. *Given the radial distance between a given curved main track and a parallel siding.*

The two tracks are to be connected by a cross-over, which shall be a reversed curve of given unequal radii beyond the frogs.

Required the central angle of each curve of the reversed curve.



Let AC = center line of inner track.

AO $= R_m$; RP $= R_1'$; RQ $= R_2'$

R_1' and R_2' are the radii of the curves beyond the frogs and may be assumed as any reasonable values.

Find a_1 and a_2 by applying the "bending process" (p. 93) and then (105) or (107).

Then in the triangle POQ find

$$\text{PO} = R_m + R_1' + a_1$$
$$\text{PQ} = R_1' + R_2'$$
$$\text{OQ} = \text{OC} + \text{CB} - \text{BQ}$$
$$= R_m + p \quad - R_2' - a_2$$

Solve for OPQ, PQO, POQ, then RQB

In practice this problem might take the following form:

Given R_m, p, g.

Assume n (or F) and n' (or F').

From these values of n and n' compute all data required for a cross-over between straight main tracks. This will involve assuming value of D_1 and D_2 and computing a_1 and a_2 by § 150 or § 151.

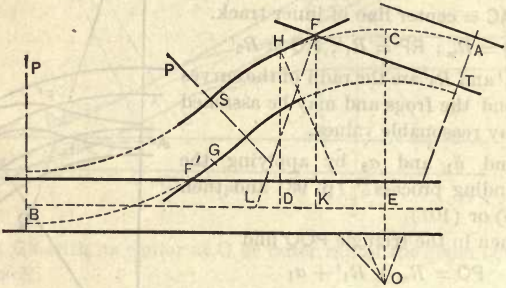
The values of a_1 and a_2 may be computed either for the case covered by (105) or by (107).

Then apply the bending process.

This will change the degrees of the turnout curves by the amount of D_m but the lengths of the turnout curves will remain unchanged (approx.) and the distances y_{a1} and y_{a2} obtained by (103) or (106) will also remain unchanged (approx.) as will also the values of a_1 and a_2.

170. Problem. *Given two main tracks not parallel. Also the unequal frog angles* F, F′ *; also* n, n', h, h', g*; also the unequal radii* R_1, R_2, *of reversed curve connecting the two from heel to heel of frogs; also the position of one frog* F.

Required the angles BPS *and* SOH *of the reversed curve; also the position of point* B.



Let OH $= R_1 + \frac{g}{2}$; BP $= R_2 + \frac{g}{2}$

HF $= h$

Set transit at theoretical point of given frog F.

Lay off FL perpendicular to TF.

Measure FL, also FLE.

Draw perpendiculars HD, FK, OC.

Let $I =$ angle between main tracks.

Then FLE $= 90° -$ LFK $= 90° - I$.

HOC $=$ HOA $-$ COA $= F - I$.

DK $= h \cos (F - I)$

FK $=$ FL $\cos I$; LK $=$ FL $\sin I$

HD $=$ FK $- h \sin (F - I)$

$$\text{CE} = \text{HD} + \left(R_1 + \frac{g}{2}\right) \text{vers } (F - I)$$

$$= \text{FL} \cos I - h \sin (F - I) + \left(R_1 + \frac{g}{2}\right) \text{vers } (F - I)$$

$$\text{KE} = \left(R_1 + \frac{g}{2}\right) \sin (F - I) - h \cos (F - I)$$

Find by (107) a_1 at A and a_2 at B.

Then $p = \text{CE} - \frac{g}{2} - a_2$

$\text{OP} = R_1 + R_2$

Solve by (76) for BPS and COS.

$\text{GPS} = \text{BPS} - F'$; $\text{SOH} = \text{COS} - (F - I)$

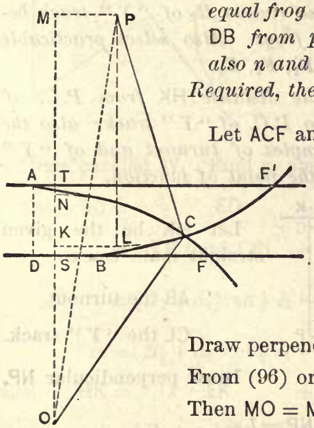
Also find $\text{BE} = (R_1 + R_2) \sin \text{BPS}$

$\text{LB} = \text{BE} - \text{KE} - \text{LK}$

From B, the point of frog F′ can readily be set in the field, its position having been established by (106) by computation.

171. Problem. *Given the R of either curve of a three-throw, or tandem split switch with switch rails of equal length; the equal frog angles F and F′; the distance* DB *from point to point of switch* $= d$; *also n and g.*

Required, the angle C of crotch frog at C.



Let ACF and BCF′ be rails of equal turnouts whose curves become parallel to DF at N and L.

Let $\text{OC} = \text{PC} = R + \frac{g}{2}$

Continue arcs to N and L; join PO.

Draw perpendiculars AD, OM, PM, KL, PL.

From (96) or (96 *A*) find $a = \text{SK} = \text{TN}$.

Then $\text{MO} = \text{MK} + \text{NO} - \text{NK}$

$$= R + \frac{g}{2} + R + \frac{g}{2} - (g - 2a)$$

$$\text{MO} \doteq 2R + 2a = 2(R + a)$$

$$\text{MP} = \text{KL} = \text{DB} = d.$$

In right triangle OMP find MOP and PO.

In isosceles triangle PCO, $\cos \text{COP} = \frac{\frac{1}{2}\text{PO}}{\text{OC}} = \frac{\frac{1}{2}\text{PO}}{R + \frac{g}{2}}$ (131)

$$2\,\text{COP} = C.$$

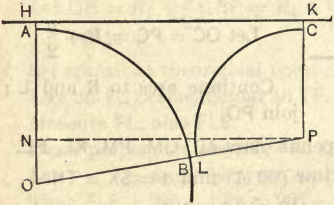
CHAPTER IX.

CONNECTING TRACKS AND CROSSINGS.

172. In many cases where a branch leaves a main track, an additional track is laid connecting the two. This is called a "*Y*" track, and the combination of tracks is called a "*Y*."

173. Problem. *Given a straight main track* HK, *also the P.C. and radius* R_1 *of curve beyond the frog. Also radius* R_2 *of "Y" track between the frogs. Also select practicable values of* F_1, F_2, F_3.

Required the distance HK *from P.C. of turnout to P.C. of "Y" track; also the central angles of turnout and of "Y" track to the point of junction.*



Let HK be the given straight main track.

AB the turnout.

CL the "*Y*" track.

Draw perpendicular NP.

Let $$\text{HK} = \text{NP} = l$$

$$\text{AOB} = I_t$$

$$\text{CPL} = I_y = 180^\circ - I_t$$

Find $\text{AH} = a_1$; $\text{KC} = a_2$; $\text{BL} = a_3$ by (107) p. 88.

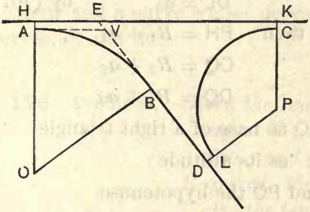
Then $$\cos \text{AOB} = \frac{\text{ON}}{\text{OP}}$$

$$\cos I_t = \frac{\text{HO} - \text{KP}}{\text{OB} + \text{BL} + \text{LP}} = \frac{R_1 + a_1 - R_2 - a_2}{R_1 + R_2 + a_3} \quad (132)$$

$$l = (R_1 + R_2 + a_3) \sin I_t \quad (133)$$

174. Problem. *Given a straight main track* HEK, *also the P.C., radius beyond the frog* OB, *and central angle* AOB, *of turnout curve connecting with a second tangent* BD; *also the radius* PC *of "Y" track. Also select practicable values of F_1, F_2.*

Required the distance HK *from P.C. of turnout to P.C. of "Y" track; also distance* BD *from P.T. of turnout curve to P.T. of "Y" track.*



Let HEK be the given main track; ABD the turnout; CL the "Y" track.

Let AO $= R_1$; CP $= R_2$

HK = AC $= l$; BD $= m$

HA $= a_1$; KC $= a_2$.

AOB $= I_1$; DL $= a_2$

CPL $= I_2$

Produce DB to E.

Draw parallel AV. Find a_1 and a_2 by (107).

$$\text{Then BD} = \text{ED} - (\text{VB} + \text{EV})$$

$$= \text{KP} \tan \tfrac{1}{2}\,\text{CPL} - \left(\text{AO} \tan \tfrac{1}{2}\,\text{AOB} + \frac{\text{HA}}{\sin \text{KEV}}\right)$$

$$= (R_2 + a_2) \tan \tfrac{1}{2} I_2 - \left(R_1 \tan \tfrac{1}{2} I_1 + \frac{a_1}{\sin I_1}\right)$$

$$m = (R_2 + a_2) \cot \tfrac{1}{2} I_1 - \left(R_1 \tan \tfrac{1}{2} I_1 + \frac{a_1}{\sin I_1}\right) \quad (134)$$

$$\text{also} \quad \text{HK} = \text{EK} + \text{EH}$$

$$l = (R_2 + a_2) \cot \tfrac{1}{2} I_1 + \left(R_1 \tan \tfrac{1}{2} I_1 - \frac{a_1}{\tan I_1}\right) \quad (135)$$

In case different frogs are used near D and K so that KC and DL are not equal, the formulas will be modified.

Let KC $= a_s$ the smaller value

DL $= a_l$ the larger value.

Following the method of § 191, p. 122:

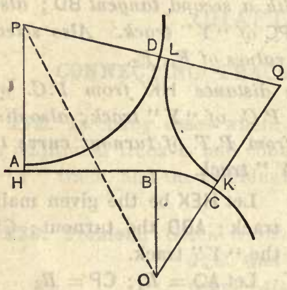
$$\text{EK} = (R_2 + a_s) \cot \tfrac{1}{2} I_1 + \frac{a_l - a_s}{\sin I_1}$$

$$\text{ED} = (R_2 + a_s) \cot \tfrac{1}{2} I_1 + \frac{a_l - a_s}{\tan I_1}.$$

175. Problem. In the accompanying sketch where

HBC = main track. AD = turnout.

LK = "*Y*" track.



Given HB = l; OB = R_m

AP = R_1; LQ = R_2.

Select F_1; F_2; F_3.

Required the points D and C.

Find AH = a_1; CK = a_2

DL = a_3 by (107)

then PH = $R_1 + a_1$

CQ = $R_2 + a_2$

DQ = $R_2 + a_3$

Considering PH + BO as base of a right triangle

HB as its altitude

Find OPH, and PO the hypotenuse

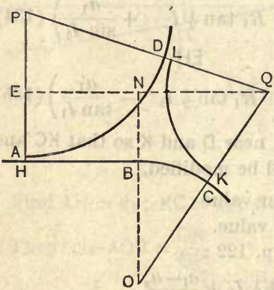
Find also PQ = $R_1 + R_2 + a_3$, QO = $R_m + R_2 + a_2$

then POQ, OPQ, PQO

then BOC, APQ

D and C will then be easily determined.

In the figure where HBC is the main track and LK is the turnout, AD the "*Y*" track.



Given OB = R_m; KQ = R_2

AP = R_1; BOC = O

Select F_1; F_2; F_3

Required the points A and D.

Find a_1, a_2, a_3 by (107)

Find QN, ON, then EP

also EQP, EQ

then EN = HB

and PQO = EQP + OQN

PQO determines position of L or D

EPQ determines length AD and EN = HB fixes H or A

In finding CK and DL in the foregoing problems of § 173 and § 175 the values of a_2 and a_3 are found from (107) after applying the "bending process."

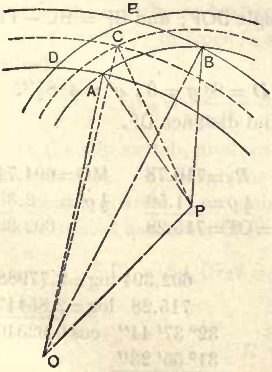
Example.

If curve AD is a 3° curve and Y track a 7° curve, then the offset $DL = a_3$ will be calculated just as it would be to connect a tangent with a 10° curve (10° = 7° + 3°), the same number or angle of frog being used.

Similarly the offset $CK = a_2$ will be the same as that for a tangent and a curve whose degree is the sum of the degrees of curves LK and BC.

176. Problem. *Given the radii, R_1, R_2, of two curves crossing at* C; *also the angle at crossing* C; *also g and g'.*

Required, the frog angles at A, B, D, E; *also the lengths on the curves of the rails* AB, BE, DE, AD.



Having given $OC = R_1$; $OCP = C$; and $PC = R_2$; find in triangle OCP, the line OP.

Having given $OA = R_1 - \frac{g}{2}$ also OP; and $PA = R_2 - \frac{g'}{2}$ find in triangle OPA, angles APO, AOP, and $OAP = A$.

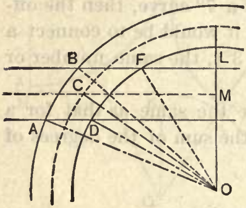
Having given $OB = R_1 + \frac{g}{2}$ also OP; and $PB = R_2 - \frac{g'}{2}$ find in triangle OPB, angles BPO, BOP, and $OBP = B$.

Then $APB = BPO - APO$, and $AB = \left(R_2 - \frac{g'}{2}\right)$ angle APB.

The frog angles at D and E, and the lengths AD, DE, EB, may be calculated in similar fashion.

177. Problem. *Given a curve crossing a tangent, R, g, g', and angle C between tangent and curve.*
Required frog angles at A, B, F, D.

Draw AO, BO, CO, FO, DO; also, MO perpendicular to CM.



Then $\quad \text{MO} = R \cos C$

$$\cos \text{MOA} = \cos A = \frac{\text{MO} - \frac{g'}{2}}{R + \frac{g}{2}} \qquad (136)$$

$$\cos \text{MOD} = \cos D = \frac{\text{MO} - \frac{g'}{2}}{R - \frac{g}{2}} \qquad (137)$$

$$\cos B = \frac{\text{MO} + \frac{g'}{2}}{R + \frac{g}{2}} \quad (138) \qquad \cos F = \frac{\text{MO} + \frac{g}{2}}{R - \frac{g}{2}} \quad (139)$$

$$\text{DOF} = \text{MOD} - \text{MOF} = D - F$$

The rail length DF $= \left(R - \frac{g}{2}\right)$ angle DOF; and BF = BL − FL.

Example. *Given* $C = 32° 28'$; $D = 8°$ $g = 3$; $g' = 4' 8\frac{1}{2}''$.
Required angle D and distance DF.

R_8 log = 2.855385
32° 28′ cos = 9.926190
MO = 604.748 log = 2.781575
$\frac{1}{2} g'$ = 2.354
607.102 log = 2.783261
OF = 715.28 log = 2.854476
31° 55′ 23″ cos = 9.928785

R_8 = 716.78
$\frac{1}{2} g$ = 1.50
OD = OF = 715.28

MO = 604.748
$\frac{1}{2} g'$ = 2.354
602.394

602.394 log = 2.779881
715.28 log = 2.854476
32° 37′ 44″ cos 9.925405
31° 55′ 23″
DOF = 0° 42′ 21″

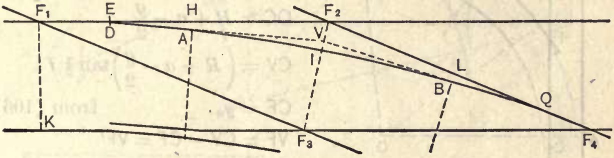
Table XX. 42′ = 0.0122173
21″ = 0.0001018
0.0123191

2.854476 = log 715.28 = $R - \frac{g}{2}$
log = 8.090579
0.945055 log = 8.812 = DF.

178. When two tracks cross at a small angle, they are often connected by a "slip switch," in which the outer rail lies entirely within the limits of the crossing and is composed of two switch rails and a connecting curve as shown in the figure below.

Problem. *Given for a crossing of two tracks the angle of crossing frog F, also n, b, g; also clearance m from actual point of frog to point of split switch; also l and t.*

Required, lengths along rail between frog points; also radius R of curve for a slip switch.



Let $DA = QB = l =$ length of switch rail

$HA = LB = t$

$F_1E = F_4Q = m =$ clearance required

Then $bn =$ distance between theoretical and actual points of frogs F_1 and F_4; in frogs F_2 and F_3 theoretical and actual points coincide.

$$F_1F_3 = \frac{g}{\sin F} + bn = F_1F_2 = F_3F_4 = F_2F_4$$

In the slip switch, produce the gauge lines DA and QB to V on the line F_2F_3. Although the point of switch has a thickness ED of about a quarter of an inch, no appreciable error results if DV be calculated assuming DF_2V to be a triangle, in which

$$F_2DV = S;\ DF_2V = 90° - \frac{F}{2};\ F_2D = F_1F_2 - m$$

Then $$AV = DV - l$$

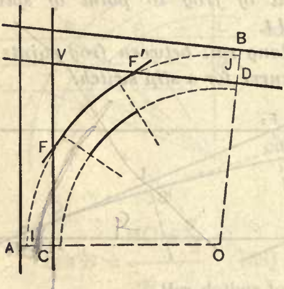
$$R + \frac{g}{2} = \frac{AV}{\tan \frac{1}{2}(F - 2\,S)}$$

Middle ordinate for chord $AB = \left(R + \frac{g}{2}\right) \text{vers } \frac{1}{2}(F - 2\,S)$

$$\text{Arc } AB = \left(R + \frac{g}{2}\right) \text{ angle } (F - 2\,S)$$

179. Problem. *Given two main tracks crossing at a given angle I; the radius R of curve connecting the two, and extending from heel of frog to heel of equal frog.*

Required the distances VF = VF′ *between actual points of frogs.*



Produce given curve to I and J where it is parallel to given main tracks.

Find by (107) $a = a'$.

$$\text{OC} = R + a - \frac{g}{2}$$

$$\text{CV} = \left(R + a - \frac{g}{2}\right)\tan \tfrac{1}{2}\, I$$

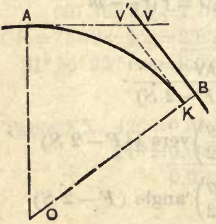
$$\text{CF} = y_a \qquad \text{from (106)}$$

$$\text{VF} = \text{CV} - \text{CF} = \text{VF}'$$

If the angle at V is at all sharp, allowance should be made for the difference between the theoretical and actual point of the frog at V.

180. Problem. *Given a straight main track* VB *and the straight line* AV *of a branch track, intersecting it at a given point* V, *and at a given angle I; also radius R of turnout curve to connect branch line and heel of frog; also F, n, h, b, g.*

Required in figure, VA, VB; *also position of point of frog.*



Find a by (107).

$$\text{AV} = \text{AV}' + \text{VV}'$$

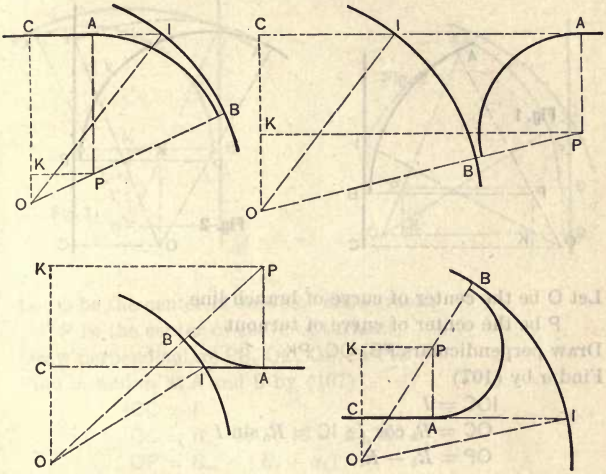
$$= R \tan \tfrac{1}{2}\, I + \frac{a}{\sin I}$$

$$\text{VB} = R \tan \tfrac{1}{2}\, I - \frac{a}{\tan I}.$$

Find F from B by fieldwork or computation, using (106).

181. Problem. *Given a curved main track* IB *of radius* R_m *a straight branch track* AI *intersecting at a given angle* I; *also radius* R_t *of turnout curve from heel of frog to branch line; also* F, n, h, b, g.

Required in the figure, IA, IOB



Let O be the center of curve of main line

P be the center of curve of turnout

Draw perpendiculars PA, OC, PK

Find a by (107)

$\text{IOC} = I$

$\text{OC} = R_m \cos I$; $\quad \text{IC} = R_m \sin I$

$\text{OP} = R_m \pm (R_t + a)$

$\text{KO} = \text{OC} \pm R_t$

$\dfrac{\text{KO}}{\text{OP}} = \cos \text{KOP}$; $\quad \text{KP} = \text{OP} \sin \text{KOP}$

$\text{IA} = \text{IC} - \text{KP}$; or $= \text{KP} - \text{IC}$; $\text{IOB} = \text{KOB} - I$ or $= I - \text{KOB}$

IOB determines position of B

Find position of frog point by (106)

182. Problem. *Given a straight main track* IBC *and a curved branch track of radius R_b intersecting at an angle I; also radius R_t of turnout curve from heel of frog to branch line; also F, n, h, b, g.*
Required in the figure, IB, IOA

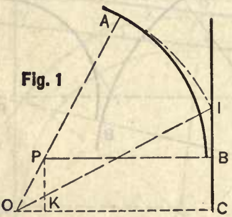


Fig. 1

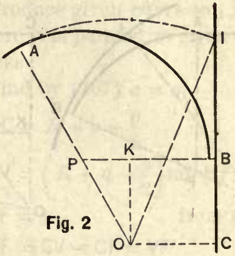


Fig. 2

Let O be the center of curve of branch line
P be the center of curve of turnout
Draw perpendiculars PB, OC, PK
Find a by (107)

$$\text{IOC} = I$$
$$\text{OC} = R_b \cos I; \quad \text{IC} = R_b \sin I$$
$$\text{OP} = R_b - R_t$$

In Figure 1

$$\text{KO} = \text{OC} - (R_t + a)$$
$$\frac{\text{KO}}{\text{OP}} = \cos \text{POK}; \quad \text{PK} = \text{OP} \sin \text{POK}$$
$$\text{IB} = \text{IC} - \text{PK}; \quad \text{IOA} = \text{POK} - I$$

In Figure 2

$$\text{PK} = (R_t + a) - \text{OC}$$
$$\frac{\text{PK}}{\text{OP}} = \sin \text{POK}; \quad \text{KO} = \text{OP} \cos \text{POK}$$
$$\text{IB} = \text{IC} - \text{KO}; \quad \text{IOA} = \text{POK} + 90° - I$$

Other cases will occur requiring figures different from those shown here · some of them will be suggested by the figures in § 181.

183. Problem. *Given a straight track and a curved track of radius R_m intersecting at a given angle I; also radius R_t of turnout curve from heel of frog to heel of frog; also F, n, h, b, g. Required in the figure,* IOA, IB

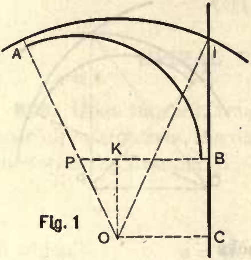


Fig. 1

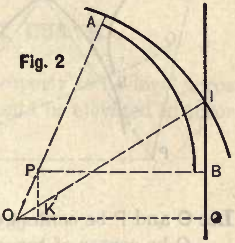


Fig. 2

Let O be the center of curve of main track
P be the center of curve of turnout
Draw perpendiculars PB, OC, OK, or PK
Find a_1 and a_2 at A and B by (107)

$$\text{IOC} = I$$
$$\text{OC} = R_m \cos I; \quad \text{IC} = R_m \sin I$$
$$\text{OP} = R_m - (R_t + a_1)$$

In Figure 1

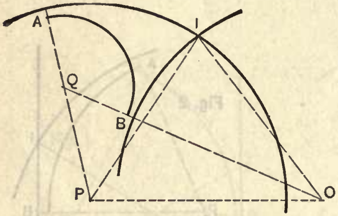
$$\text{PK} = R_t + a_2 - \text{OC}$$
$$\frac{\text{PK}}{\text{OP}} = \sin \text{POK}; \quad \text{KO} = \text{OP} \cos \text{POK}$$
$$\text{IB} = \text{IC} - \text{KO}; \quad \text{IOA} = \text{POK} + 90° - I$$

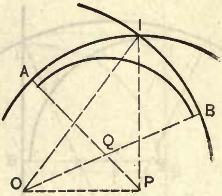
In Figure 2

$$\text{KO} = \text{OC} - (R_t + a_2)$$
$$\frac{\text{KO}}{\text{OP}} = \cos \text{POK}; \quad \text{PK} = \text{OP} \sin \text{POK}$$
$$\text{IB} = \text{IC} - \text{PK}; \quad \text{IOA} = \text{POK} - I$$

Other cases will occur requiring figures different from those shown here; some of them will be suggested by the figures in § 181.

184. Problem. *Given two curved lines of track of radii R_1 R_2 crossing each other, intersecting at an angle I; also the radius R_t of turnout from heel to heel of frog; also F, n, h, b, g*
Required in the figure, API, IOB





Let O and P be centers of main tracks
Q be center of turnout

$$OIP = I$$

Find a_1 at A, and a_2 at B by (107)

In triangle IOP, $IO = R_2$; $IP = R_1$; $OIP = I$

Solve for OP, IOP, IPO

In triangle OQP, $QP = R_1 - (R_t + a_1)$
$QO = R_2 \pm (R_t + a_2)$
OP computed

Solve for QOP, QPO, OQP
From QPO and IPO, find API
From IOP and QOP, find IOB

CHAPTER X.

SPIRAL EASEMENT CURVE.

185. Upon tangent, track ought properly to be level across; upon circular curves, the outer rail should be elevated in accordance with the formula

$$e = \frac{gv^2}{32.2\,R}$$

in which e = elevation in feet

g = gauge of track

v = velocity in feet per second

R = radius of curve in feet

In passing around a curve, the centrifugal force $c = \dfrac{Wv^2}{32.2\,R}$

It is desirable for railroad trains that the centrifugal force should be neutralized by an equal and opposite force, and for this purpose, the outer rail of track is elevated above the inner. Any pair of wheels, therefore, rests upon an incline, and the weight W resting on this incline may be resolved into two components, one perpendicular to the incline, the other parallel to the incline, and towards the center of the curve.

The component p parallel to the incline will be $p = \dfrac{We}{g}$

It will be a very close approximation to assume that c acts parallel to the incline (instead of horizontally). The centrifugal force will be balanced (approx.) if we make

$$p = c \text{ or } \frac{We}{g} = \frac{Wv^2}{32.2\,R}$$

whence

$$e = \frac{gv^2}{32.2\,R} \qquad (140)$$

In passing directly from tangent to circular curve, there is a point (at *P.C.*) where two requirements conflict; the track cannot be level across and at the same time have the outer rail elevated. It has been the custom to elevate the outer rail on the tangent for perhaps 100 feet back from the *P.C.* This is unsatisfactory. It has therefore become the best practice to introduce a curve of varying radius, in order to allow the train to pass gradually from the tangent to the circular curve.

186. The transition will be most satisfactorily accomplished when the elevation e increases uniformly with the distance l from the *T.S.* (point of spiral) where the spiral easement curve leaves the tangent; then $\frac{e}{l}$ is a constant

$$\text{or} \quad \frac{gv^2}{32.2\,Rl} = A \text{ (a constant) or } Rl = \frac{gv^2}{32.2\,A}$$

Since g, v, A are constants, $Rl = C$ (a constant)

Then $$Rl = R_c l_c \text{ and } R = \frac{R_c l_c}{l} \tag{141}$$

also $$\frac{l}{D} = \frac{l_c}{D_c} \text{ or } \frac{D}{D_c} = \frac{l}{l_c} \text{ (approx.)} \tag{141 A}$$

where R_c = radius of circle

D_c = degree of circular curve

l_c = total length of spiral in feet.

Let s = the "Spiral Angle" or total inclination of curve to tangent at any point.

s_c = spiral angle where spiral joins circle.

Then $$Rds = dl \text{ or } ds = \frac{dl}{R}$$

from (141) $$= \frac{ldl}{R_c l_c}$$

$$s = \frac{l^2}{2\,R_c l_c} \tag{142}$$

Again $dx = dl \sin s$ and $dy = dl \cos s$

All values of s will generally be small, and we may assume

$$\sin s = s \qquad \text{and} \qquad \cos s = 1$$

then

$$dx = s\,dl \qquad\qquad dy = dl$$

$$= \frac{l^2\,dl}{2\,R_c l_c} = \frac{y^2\,dy}{2\,R_c l_c} \qquad\qquad y = l$$

Integrating, $$x = \frac{y^3}{6\,R_c l_c} \tag{143}$$

which is the equation of the "Cubic Parabola," a curve frequently used as an easement curve.

If, however, the approximation $\cos s = 1$ be not used, the resulting curve will be more nearly correct than is the Cubic Parabola. In this case $\sin s = s$

$$dx = s\,dl = \frac{l^2\,dl}{2\,R_c l_c}$$

Integrating, $$x = \frac{l^3}{6\,R_c l_c} \tag{144}$$

The resulting curve we may call, for the lack of a better name, the "Cubic Spiral" Easement Curve.

The Cubic Parabola is well adapted to laying out curves by "offsets from the tangent." Modern railroad practice favors "deflection angles" as the method of work wherever practicable. In the case of an easement curve the longitudinal measurements are most conveniently made as chords along the curve, so that $x = \frac{l^3}{6\,R_c l_c}$ represents a curve more convenient for use than is $x = \frac{y^3}{6\,R_c l_c}$ as well as more nearly correct. Evidently the properties of the two curves will be very similar.

The following notation in connection with spirals has been adopted by the Am. Ry. Eng. Ass'n. For the point of change

from tangent to spiral, *T.S.*
from spiral to circular curve, *S.C.*
from circular curve to spiral, *C.S.*
from spiral to tangent, *S.T.*

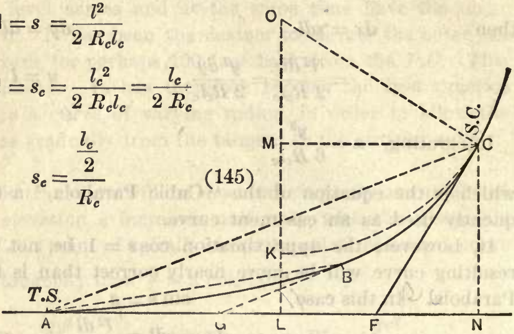
This notation will be adopted here.

187. *Given, in a Cubic Spiral,* l, l_c, R_c
Required s, s_c, *and "total deflection angles"* i, i_c

(142) $\text{BGN} = s = \dfrac{l^2}{2\,R_c l_c}$

and $\text{CFN} = s_c = \dfrac{l_c^{\,2}}{2\,R_c l_c} = \dfrac{l_c}{2\,R_c}$

$$s_c = \frac{\frac{l_c}{2}}{R_c} \qquad (145)$$



This (145) is the expression (in the form of length of arc for radius 1) for the central angle of the connecting circular curve for a length of one-half the length of spiral. In another form it is

$$s_c = \frac{l_c D_c}{200} \quad (l_c \text{ in feet and } s_c \text{ in degrees}) \qquad (145\,A)$$

If the circular curve be produced back from C to K where it becomes parallel to AN, its length in feet will be $\frac{l_c}{2}$ since KOC = CFN $= s_c$.

Also $\text{AL} = q = \frac{l_c}{2}$ (approx.) (145 *B*)

Again for any point B on the spiral

$$\sin \text{BAN} = \sin i = \frac{x}{l} \text{ (approx.)}$$

$$i = \frac{x}{l} \text{ (approx.)} = \frac{l^3}{6\,R_c l_c l} = \frac{l^2}{6\,R_c l_c}$$

But $s = \dfrac{l^2}{2\,R_c l_c}$ from (142)

Whence $i = \dfrac{s}{3}$ and $i_c = \dfrac{s_c}{3}$ (146)

Also $i : i_c = l^2 : l_c^{\,2}$; or $i = i_c \left(\dfrac{l}{l_c}\right)^2$ (146 *A*)

Also the back deflection ABG = BGN − BAN

$$= s - i = 3\,i - i = 2\,i$$

Also $\text{ACF} = 2\,i_c$ (146 *B*)

It will be observed that the Cubic Spiral has the following properties (some slightly approximate):

(*a*) The degree of curve varies directly with the length from the *T.S.* (141 *A*)

(*b*) The deflection angles vary as the squares of the lengths from the *T.S.* (146 *A*)

(*c*) The offsets from the tangent vary as the cubes of the lengths from the *T.S.* (144)

(*d*) The "spiral angle" at the point where the spiral joins the circular curve is equal to the central angle of a circular curve of the same degree and of a length one-half that of the spiral. (145)

(*e*) The deflection angle to any point on the spiral is one-third the spiral angle at that point. (146)

188. *Given l, l_c, R_c. Required y and y_c.*

From (30) the excess of hypotenuse over base

$$e = c - a = \frac{h^2}{2c}$$

Then in the Cubic Spiral, at any point on the spiral, let the

excess $\qquad de = dl - dy$

from (30) $\qquad de = \dfrac{dx^2}{2\,dl} = \dfrac{l^4\,dl^2}{2 \times 4\,R_c^2 l_c^2 dl} = \dfrac{l^4\,dl}{8\,R_c^2 l_c^2}$

integrating, $\qquad e = l - y = \dfrac{l^5}{40\,R_c^2 l_c^2}$

$$y = l - \frac{l^5}{40\,R_c^2 l_c^2} \qquad (147)$$

$$y_c = l_c - \frac{l_c^5}{40\,R_c^2 l_c^2} = l_c - \frac{l_c^3}{40\,R_c^2} \qquad (147\,A)$$

189. *Given R_c, y_c, x_c, s_c. Required* AL $= q$ and LK $= p$.

CN $= x_c$ and AN $= y_c$

AL = AN − OC sin COK or $q = y_c - R_c \sin s_c$ (148)

LK = CN − OC vers COK $\quad p = x_c - R_c \operatorname{vers} s_c$ (148 *A*)

Tables have been computed for the Cubic Spiral described above. These have been abandoned in favor of the spiral adopted by the Am. Ry. Eng. Ass'n, and new tables arranged for this spiral which is described in the following section.

190. In the Cubic Spiral, the lengths have been considered as measured along the curve itself; but measurements in the field are necessarily taken by chords. This is recognized in defining the degree of a simple curve § 39 as the angle at the center subtended by a *chord* of 100 ft. Consistent with this, in the Am. Ry. Eng. Ass'n Spiral, the length of spiral is measured by *ten equal chords*, so that the theoretical curve is brought into harmony with field practice. This spiral will be referred to here as the A. R. E. A. Spiral, and adopted in place of the Cubic Spiral. The two curves substantially coincide up to the point where $s_c = 15°$, and the discussion of the Cubic Spiral applies in a general way to the A. R. E. A. Spiral also. Beyond $s_c = 15°$ the A. R. E. A. Spiral has its tables computed substantially without approximations, making it a very perfect and convenient transition curve even for sharp curves on street railways.

The A. R. E. A. Spiral retains the following features characteristic of the Cubic Spiral:

(*a*) The degree of curve varies directly with the length from the *T.S.*

$$\frac{D}{D_c} = \frac{l}{l_c} \qquad (141\,A)$$

(*b*) The deflection angles vary as the squares of the lengths from the *T.S.*

$$\frac{i}{i_c} = \left(\frac{l}{l_c}\right)^2 \qquad (146\,A)$$

(*d*) The spiral angle at the point where the spiral joins the circular curve is equal to the central angle of a circular curve of the same degree and of a length one-half that of the spiral.

$$s_c = \frac{l_c D_c}{200} \qquad (145\,A)$$

(*e*) For practical purposes the deflection angle to any point on the spiral is one-third the spiral angle at the point (up to a value of $s_c = 15°$), or

$$i = \frac{s}{3} \qquad (146)$$

Beyond 15° and up to 45° for values of s_c, correct values computed by the Am. Ry. Eng. Ass'n show the following empirical formula to apply:

$$i = \frac{s}{3} - 0.00297\, s^3$$

i and s are in degrees. $0.00297\, s^3$ gives results in *seconds*.

With the A.R.E.A. Spiral, the angle made with the tangent at the *T.S.* by the first chord is taken as

$$\alpha_1 = \frac{s_c}{300}$$

No appreciable error is found to result if the angles made by successive chords with this tangent are taken as exact multiples of α_1 as follows:

1, 7, 19, 37, 61, 91, 127, 169, 217, 271

It is evident that these values of α_1, α_2, etc. depend upon s_c and are independent of the length of chord used.

For computing values of x_c, y_c the method of "offsets from the tangent" § 66 is adopted and co-ordinates x, y, at each chord point are found by using

$$\frac{l_c}{10} \sin \alpha_1, \ \frac{l_c}{10} \cos \alpha_1 ; \quad \frac{l_c}{10} \sin \alpha_2, \ \frac{l_c}{10} \cos \alpha_2, \text{ etc.}$$

For a given value of s_c the final co-ordinates $x_c y_c$ will be directly proportional to l_c, so that $\frac{x_c}{l_c} \cdot \frac{y_c}{l_c}$ will be constants of a given value of s_c. It will be true of the long chord C from *T.S.* to *S.C.* that $\frac{C}{l_c}$ will also be a constant.

A condensed table of values of $\frac{x_c}{l_c}, \frac{y_c}{l_c}, \frac{C}{l_c}$ is given in Table VII, B; for values of s_c differing by $0° \, 30'$.

This table will have occasional rather than frequent use; intermediate values may be interpolated with sufficient precision for ordinary cases; the labor of interpolating will not be burdensome.

From these values of x_c and y_c, determined as above, values of i_c have been computed for successive values of s_c up to 45° and these are tabulated in Table VII. All of the computations mentioned above have been made by the Am. Ry. Eng. Ass'n.

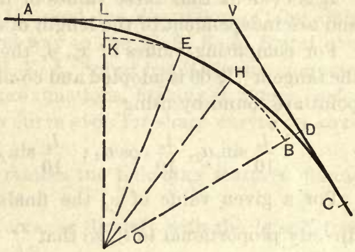
For convenient use in the field the deflection angle to each chord point is necessary, and the author has computed these for successive values of s_c and tabulated them in Table VII.

The deflection angles are constant for a given value of s_c and may be used for this value of s_c whatever the length of spiral, provided the chord length is made one-tenth the length of spiral.

Values of p and q have been computed by the author by (148, and (148 A) for various degrees of curve, and for various lengths of spiral, and these are found in Table VI which gives for each degree and half degree of curve, a series of lengths of spiral, and for each length, values of s_c, p, q, x_c, y_c, C.

191. Problem. *Given I, l_c, and R_c or D_c.*

Required the Tangent Distance T_s.



Find q and p by § 189 or by Table VI or by Table VII B.

(a) When the spirals at both ends of the circular curve are alike.

Let AL $= q$ and LK $= p$

AV $=$ AL $+$ LV

$= $ AL $+$ OL $\tan \frac{1}{2}$ LOD

$T_s = q + (R_c + p) \tan \frac{1}{2} I$

$T_s = q + T_c + p \tan \frac{1}{2} I$ (149)

where T_c is tangent distance for circular curve alone, for the given value of I.

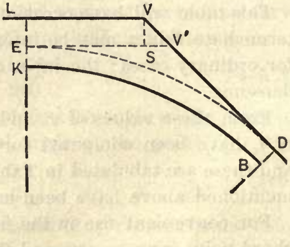
(b) When different spirals are used at the ends, separate values must be found for LV and DV.

Let LK $= p_l$

BD $= p_s$

Draw arc DE.

Also perpendiculars EV′, VS.



$$\text{VS} = p_l - p_s$$

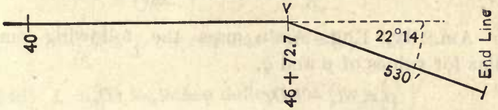
$$\text{VV}' = \frac{p_l - p_s}{\sin I}$$

$$\text{SV}' = \frac{p_l - p_s}{\tan I}$$

$$\text{LV} = (R + p_s) \tan \tfrac{1}{2} I - \frac{p_l - p_s}{\tan I} \qquad (149\ A)$$

$$\text{DV} = (R + p_s) \tan \tfrac{1}{2} I + \frac{p_l - p_s}{\sin I} \qquad (149\ B)$$

Example. *Given a line as shown in sketch.*
Required to connect the tangents by a 4° curve with a spiral 180 feet long at each end.



Find T_c Table III. 22° 14′ $T_1 = \underline{1125.8(4°}$

281.45

Table IV. $\underline{.05}$ corr.

Table VI. $p = 0.94$; $q = 89.97$ $281.50 = T_c$

nat tan $\frac{1}{2}$ (22° 14′) = 0.19649 $89.97 = q$

$0.19649 \times 0.94 = 0.18$ $\underline{.18 = p \tan \frac{1}{2} I}$

$371.65 = T_s$

$s_c =$ 3° 36′
$2\,s_c =$ 7° 12′
$I =$ $\underline{22° 14′}$
$I - 2\,s_c =$ 15° 02′

$L = \dfrac{4°)15.0333}{375.8}$

Table VI. $s_c = 3° 36′ = 3°.6$
Deflection angles for spiral from
Table VII. for $s_c = 3°.6$

Transit at	43 + 01.0 *T.S.*
$i = 0° 01′$ to	43 + 19.0
0° 03′	43 + 37.0
0° 06′	43 + 55.0
0° 11′	43 + 73.0
0° 18′	43 + 91.0
0° 26′	44 + 09.0
0° 35′	44 + 27.0
0° 46′	44 + 45.0
0° 58′	44 + 63.0
1° 12′	44 + 81.0

$V = 46 + 72.7$
$\underline{3 + 71.7 = T_s}$
T.S. 43 + 01.0
$\underline{1 + 80.0 = l_c}$
S.C. 44 + 81.0
$\underline{3 + 75.8 = L}$
C.S. 48 + 56.8
$\underline{1 + 80.0 = l_c}$
S.T. 50 + 36.8

Defl. angles for circular curve with transit at 44 + 81.0

$s_c = 3° 36$
$i_c = \underline{1° 12}$
back deflection to *T.S.* $= 2° 24$

for $c_i = 19$, $\dfrac{d_i}{2} = 0° 23′$ 45
2° 23′ 46
4° 23′ 47
6° 23′ 48

for $c_f = 56.8$, $\dfrac{d_f}{2} = \dfrac{1° 08′}{7° 31′}$ 48+56.8

$$\frac{I - 2\,s_c}{2} = \frac{15° 02′}{2} = 7° 31′ \quad \textit{Check}$$

192. Problem. *Given D_c and l_c.*
Required p, q, and other data for spiral

$$\text{from (145 } A) \quad s_c = \frac{l_c D_c}{200}. \qquad (145\,A)$$

The Am. Ry. Eng. Ass'n uses the following empirical formulas for values of p and q,

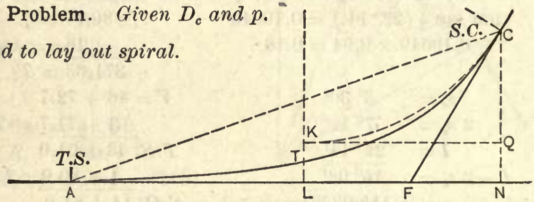
$$p = al_c - bD_c \qquad q = el_c - fD_c.$$

Tables of the coefficients a, b, e, f, condensed from the A. R. E. A. Tables are given in Table VII B for values of s_c differing by 30′; intermediate values may be interpolated.

The deflection angles may be found as before from Table VII.

193. Problem. *Given D_c and p.*

Required to lay out spiral.



from (145) $\quad \mathsf{KC} = \frac{l_c}{2}$ (approx.),

from (145 B) $\quad q = \frac{l_c}{2}$ (approx.)

$$\mathsf{CN} = x_c = \frac{l_c^3}{6\,R_c l_c} = \frac{l_c^2}{6\,R_c} \qquad \text{for spiral}$$

from (26) $$\mathsf{CQ} = \frac{c^2}{2\,R} = \frac{\left(\frac{l_c}{2}\right)^2}{2\,R_c} = \frac{l_c^2}{8\,R_c} \quad \text{(approx.)} \quad \text{for circle}$$

therefore $$\mathsf{CN} = \frac{4}{3}\mathsf{CQ} = \mathsf{CQ} + \frac{\mathsf{CQ}}{3} = \mathsf{CQ} + \mathsf{QN} \quad \text{(approx.)}$$

$$\mathsf{CQ} = 3\,\mathsf{QN} = 3\,\mathsf{KL} = 3\,p \quad \text{(approx.)}$$

$$\mathsf{CN} = 4\,\mathsf{QN} = 4\,\mathsf{KL} \qquad \text{(approx.)}$$

from (144) $$\frac{x}{x_c} = \frac{l^3}{l_c^3} \qquad (149\,C)$$

$$\mathsf{CN} = 2^3\mathsf{TL} = 8\,\mathsf{TL} = 4\,\mathsf{KL}$$

$$\mathsf{TL} = \frac{\mathsf{KL}}{2} = \frac{p}{2} \quad \text{(approx.)} \qquad (149\,D)$$

From CQ $= 3p$ the length of curve may be readily determined. If circular curve KC has center at O, KOC = CFN $= s_c$.

$$\text{vers KOC} = \frac{\text{CQ}}{\text{OK}} \quad \text{or} \quad \text{vers } s_c = \frac{3p}{R_c}$$

$$\frac{100\,s_c}{D_c} = L \text{ for circular curve KC ; } l_c = 2L$$

from (146) $i_c = \frac{s_c}{3}$; for other deflections $i = i_c\left(\frac{l}{l_c}\right)^2$ (146 *A*)

The back deflection ACF $= 2\,i_c$.

Table XXXIII will facilitate some of these computations.

By the above method, the values of s_c and l_c may be reached with substantial accuracy without the use of the spiral tables. Where close results are necessary, p may be re-computed by Table VII *B* from the values of s_c and l_c already found by the above formulas. If the new value of p is not sufficiently close to the given value, correct values of s_c and l_c may be found by trial. The value of q is found by Table VII *B*.

The deflection angles may then be taken from Table VII.

While the method of § 193 is more laborious than the more common method of § 191, it has special value because it is thoroughly elastic and any given length of spiral may be used. In a similar way, if the value of p (together with D_c) determines the spiral to be used, the method of § 193 becomes useful.

Approximate Method.

Problem. *Given D_c and either l_c or p.*

Required s_c and the deflection angles without using tables.

Assume the long chord KC to be equal to $\frac{l_c}{2}$.

$$R_1 = 5730 \qquad R_a = \frac{5730}{D_a}$$

By § 193 find $3p$ from R_c and L by (26) ; or find L from R_c and $3p$ by (26)

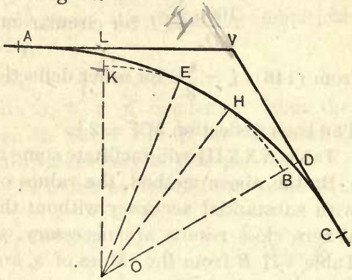
$$L = q \text{ (approx.) ; } s_c = \frac{l_c D_c}{200}\text{; and } i = \frac{s_c}{3}$$

Other deflections are found by $i = i_c\left(\frac{l}{l_c}\right)^2$ (146 *A*)

Computations involving the use of (26) may be made using Tables XXXIII and XXXIV.

194. Fieldwork of Laying out Spiral.

(a) Select on the ground the vertex V and measure I; or else fix on ground, point L opposite the point K where the circular curve will become parallel to tangent.



(b) Select the length l_c of spiral to join given circular curve; this may be taken from Table VI or computed by § 193 from D_c and p.

(c) Find value of q and s_c from Table VI or by § 193.

(d) Set $T.S.$ at A by measuring T_s from vertex, or by measuring q from point L, as the case may be.

(e) With transit at $T.S.$ run in spiral using deflection angles from Table VII.

(f) With transit at $S.C.$ turn vernier to 0° and beyond 0° to measure angle $s_c - i_c$ (this will be $2\,i_c$ when s_c is less than 15°).

(g) Take backsight on $T.S.$, and when vernier reads 0° the line of sight is on auxiliary tangent.

(h) Run in circular curve by deflection angles; the central angle of circular curve $= I - 2\,s_c$.

(i) With transit at $S.T.$ (*not* at $C.S.$) run in second spiral.

(k) "Check" on $C.S.$

(l) If the "check" is not substantially perfect, *re-set the point at C.S.*

It is important that each spiral shall be correct throughout its entire length. In case the spiral and circular curve do not check properly at the $C.S.$, the discrepancy should be thrown into the circular curve where its effect will be unimportant.

When the circular curve is visible from the $C.S.$ the general method of § 62 will give the best results, as follows:

(A) Lay out first spiral from $T.S.$ to $S.C.$

(B) Lay out second spiral from $S.T.$ to $C.S.$

(C) Set up transit at $C.S.$ and lay out circular curve from $S.C.$ to $C.S.$ and check angle to $S.T.$

195. *Given D_c and l_c.*

Required to lay out spiral by offsets from the tangent.

From Table VI find value of x_c.

Find other values of x at convenient intervals by formula

$$x = x_c \left(\frac{l}{l_c} \right)^3 \qquad \text{(from 144)}$$

This method will be useful at times but more often spirals will be laid out by deflection angles.

Example. *Given $D_c = 4°$, $l_c = 240$.*

Required offsets from tangent to spiral.

Take offsets at middle, quarter, and eighth points.

Table VI gives;

for	$l_c = 240$	$x_c = 6.70$
at	$l_4 = 120$	$x_4 = 6.70 \div 8 = 0.8375$
	$l_2 = 60$	$x_2 = 0.8375 \div 8 = 0.1047$
	$l_1 = 30$	$x_1 = 0.1047 \div 8 = 0.0131$
	$l_3 = 90$	$x_3 = 0.0131 \times 3^3 = 0.35$
	$l_5 = 150$	$x_5 = 0.0131 \times 5^3 = 1.64$
	$l_6 = 180$	$x_6 = 0.0131 \times 6^3 = 2.83$
	$l_7 = 210$	$x_7 = 0.0131 \times 7^3 = 4.49$

The "cubic spiral" will be laid out by measuring successive chords of 30 ft. each, and measuring the proper offset from the tangent.

For the "Cubic Parabola,"

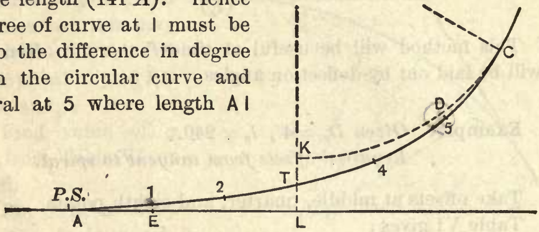
the formula is $$x = \frac{y^3}{6\,C}$$

whence $$x = x_c \left(\frac{y}{y_c} \right)^3$$

The computations may be the same as for the cubic spiral. The successive distances of 30 will be laid off on the tangent and the offset laid off at right angles to the tangent.

196. It may occasionally (although not frequently) happen that the entire spiral cannot be laid out from the *T.S.*, and it will be necessary to determine deflection angles when the transit is at some intermediate point on the spiral. It will be desirable to occupy some regular chord point.

In any Cubic Spiral, the degree of curve *D* increases uniformly with the length (141 *A*). Hence the degree of curve at 1 must be equal to the difference in degree between the circular curve and the spiral at 5 where length A 1 = C 5.



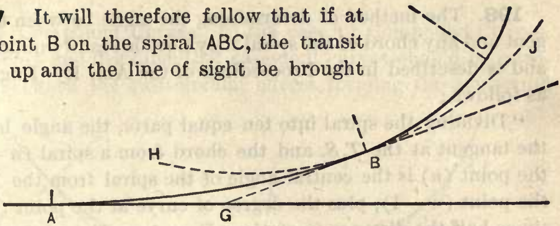
Since the divergence in the degree of the spiral is the same for a given distance, whether this divergence be from the tangent AL or from the curve CK, it will naturally follow from the principles established in § 69, that the offset to the spiral for a given distance from C will be the same as the offset for the same distance from A, since the change in degree at corresponding points is always the same whether from tangent or curve.

The same conclusion will be reached by referring to § 160 near the bottom of p. 93, where the elastic model and the "bending process" is referred to; this bending process being there found to be correct (approx.) from the demonstration § 158, p. 92. If this principle be correct, it will follow that KT = TL, which may be considered an extreme case. That KT = TL is demonstrated (in 149 *D*) to be correct is an additional assurance of the correctness of the principle stated above.

It will further follow if E 1 and D 5 are equal, and at equal distances from A and C respectively, that the angles E A 1 and D C 5 will be equal (closely). For the offset divided by the distance gives approximately the sine of the angle, and since the sines are equal, the angles also are equal; similarly the angles LAT and KCT are equal.

In other words, the divergence of any given spiral for a given distance, is the same either in offset or in angle, whether the divergence be from the tangent or from the circular curve.

197. It will therefore follow that if at any point B on the spiral ABC, the transit be set up and the line of sight be brought



on the auxiliary tangent BG at that point, then the deflection angle to any forward point on the spiral will be the sum of (1) the "total deflection angle," for the distance from B to that point, due to the circular curve HBJ, whose degree is the degree of the spiral at B; and (2) the "total deflection angle" from the original tangent for that spiral for the same distance reckoned from the *T.S.* For any back point, the deflection angle from this auxiliary tangent will be the difference between these angles.

The proper use of these deflection angles will allow the line of sight to be brought on the auxiliary tangent, as well as give means for setting all points on the spiral.

Example. Required forward deflection angles from point 6 on a spiral 300 feet long, to join 5° curve.

$$s_c = 7^\circ\,30' = 7^\circ.5$$

The tangent BG is found by laying off from chord AB, twice the forward deflection to point 6, or $2 \times 54' = 1^\circ\,48'$.

D at point $6 = 0.6 \times 5^\circ$			$= 3^\circ\,00'$
Deflection angle for 30 ft. on 3° curve			$= 27'$
The total angles will be at point	7,	$27' + 01'$	$= 28'$
	8,	$54' + 06'$	$= 1^\circ\,00'$
	9,	$81' + 13'$	$= 1^\circ\,34'$
	10,	$108' + 24'$	$= 2^\circ\,12'$
The back deflections will be at point	5,	$27' - 01'$	$= 26'$
	4,	$54' - 06'$	$= 48'$
	3,	$81' - 13'$	$= 1^\circ\,08'$
	2,	$108' - 24'$	$= 1^\circ\,24'$
	1,	$135' - 37'$	$= 1^\circ\,38'$
	0,	$162' - 54'$	$= 1^\circ\,48'$

The back deflection from point 6 to *T.S.* also $= 0^\circ\,54' \times 2 = 1^\circ\,48'$.

198. The method of determining the angle between the tangent and any chord of the spiral may now be readily understood, and is described in the Proceedings of the Am. Ry. Eng. Ass'n as follows:

"Dividing the spiral into ten equal parts, the angle between the tangent at the *T.S.* and the chord from a spiral $(n-1)$ to the point (n) is the central angle of the spiral from the *T.S.* to the point $(n-1)$, plus the degree of curve at the point $(n-1)$ times half the distance in stations from $(n-1)$ to (n), plus the deflection from the tangent at the *T.S.* to the chord subtending the first tenth of the spiral"

or
$$\alpha_n = \left(\frac{n-1}{10}\right)^2 s_c + \frac{n-1}{100} s_c + \frac{s_c}{300}$$
$$= \frac{3n^2 - 3n + 1}{300} s_c$$

"Substituting the successive numerals 1 to 10 for n, the successive values" of α "are 1, 7, 19, 37, 61, 91, 127, 169, 217, and 271 — 300ths" of s_c.

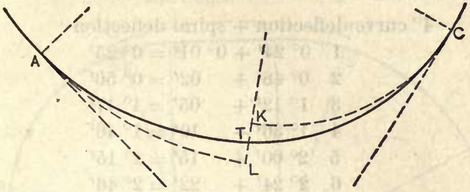
In a similar fashion the Am. Ry. Eng. Ass'n has calculated the forward and backward deflections when the transit is at an intermediate station on the spiral and Table VII *A* shows these as multiples (by full numbers) of the first chord deflection angle i_1.

In finding the numbers for this Table the assumption was made that the deflection angle from the *T.S.* to any point is one third the spiral angle to that point, which is approximate only where s_c exceeds 15°. When the transit is set at a point P' and a deflection angle (from the auxiliary tangent at P') is taken to another point P'' the Am. Ry. Eng. Ass'n states:

"The formulas and rule are approximate and should not be used when the central angle from P' to P'' exceeds the central angle from the *T.S.* by more than 15°."

Table VII *A* furnishes a very simple method of finding forward and back deflections when it becomes necessary to set the transit at an intermediate point on the spiral. While multiplying i_1 may be somewhat burdensome, setting up at intermediate points will not be frequent, and simplicity is of prime importance.

199. Compound Curves. In the case of Compound Curves, it is proper and desirable that easement curves should be introduced between the two circular curves forming the compound curve.



Problem. *Given in a Compound Curve, D_l, D_s, p, or l.*
Required the Deflection Angles for a Cubic Spiral to connect the circular curves.

(*a*) Find by Table VII or by § **193** the Deflection Angles proper for a Cubic Spiral to connect a tangent with a circular curve of degree $= D_l - D_s$.

Let these $= i_1, i_2, i_3$, etc.

(*b*) Find the deflection angles to corresponding points on one of the circular curves, the auxiliary tangent for these being at the point where the Cubic Spiral leaves this circular curve (where the transit will be set).

Let these $= \frac{d_1}{2}, \frac{d_2}{2}, \frac{d_3}{2}$, etc.

(*c*) The required total deflections from A will be for

point 1 $\quad \frac{d_1}{2} + i_1 \qquad$ point 2 $\quad \frac{d_2}{2} + i_2$

point 3 $\quad \frac{d_3}{2} + i_3$ etc.

The required total deflections from C will be for

point 1 $\quad \frac{d'_1}{2} - i_1 \qquad$ point 2 $\quad \frac{d'_2}{2} - i_2$ etc.

Similar procedure may be followed if it be desired to lay out the spiral by offsets. Convenient points may be set on the circular curves and the offsets taken from either curve.

Example. *Given* $D_l = 4°$, $D_s = 7°$, $l_c = 200$.

From Tables VI and VII find deflection angles for a curve of $D = 7° - 4° = 3°$ with $l_c = 200$, where $s_c = 3° 00'$. On 4° circular curve deflection angle for 20′ chord = 0° 24′.

		4° curve deflection + spiral deflection
for point	1	0° 24′ + 0° 01′ = 0° 25′
	2	0° 48′ + 02′ = 0° 50′
	3	1° 12′ + 05′ = 1° 17′
	4	1° 36′ + 10′ = 1° 46′
	5	2° 00′ + 15′ = 2° 15′
	6	2° 24′ + 22′ = 2° 46′
	7	2° 48′ + 29′ = 3° 17′
	8	3° 12′ + 38′ = 3° 50′
	9	3° 36 + 49′ = 4° 25′
	10	4° 00′ + 60′ = 5° 00′

These are total deflection angles from auxiliary tangent when the transit is on the 4° curve.

Field work.

(*a*) Fix L or K in ground from topography or other practical requirements, the same as for any compound curve.

(*b*) Assume l_c and compute p.

(*c*) Fix A and C, true transit points on curve at distances $\frac{l_c}{2}$ from L or K.

(*d*) Set transit at A.

(*e*) Bring line of sight on auxiliary tangent at A.

(*f*) Set off "total deflection" angles to spiral and run in spiral.

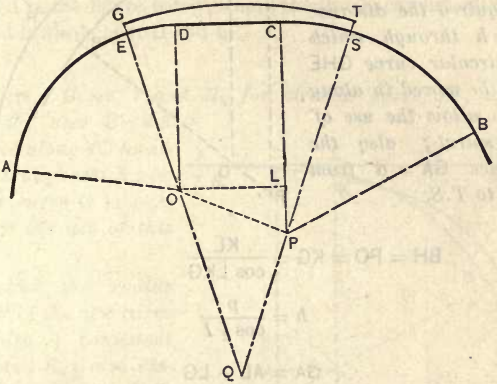
200. Determination of Length of Spiral.

The basis used by the Am. Ry. Eng. Ass'n for fixing the proper length of spiral is the increase per second of the elevation of the outer rail. Too rapid an increase, it is thought, will cause some discomfort to passengers. The discussion is too extended for a pocket book, and will not be attempted here.

The Am. Ry. Eng. Ass'n has prepared a diagram shown as Table VII *C* which covers the recommendation of the Association for fixing the length of spirals.

201. Problem. *Given two simple curves with connecting tangent.*

Required to substitute a simple curve of given radius with connecting spirals at each end.



Let DC $= t =$ given tangent, connecting the two curves AD and CB of radii R_s and R_l respectively.

Let GT be the given new curve of radius R_c.

Assume suitable spirals and find from table VI, GE $= p_1$ and ST $= p_2$ for these spirals, also q_1 and q_2.

Join OP and draw perpendicular OL.

Then $\tan \text{LOP} = \dfrac{R_l - R_s}{t}$; $\text{OP} = \dfrac{t}{\cos \text{LOP}}$

In the triangle OPQ there are given

$$\text{OP} = \frac{t}{\cos \text{LOP}}; \ \text{OQ} = R_c - R_s - p_1 \ ; \ \text{QP} = R_c - R_l - p_2$$

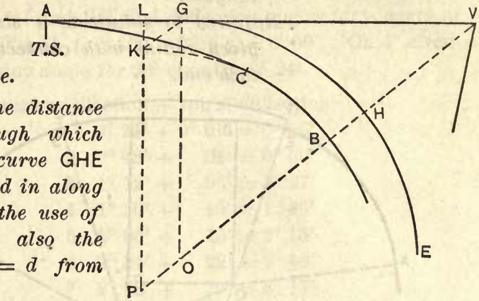
Solve this triangle for OQP, QOP, OPQ.

Then $\text{CPS} = 180° - (\text{OPQ} + \text{OPL})$

$\text{EOD} = \ 90° - (\text{QOP} + \text{LOP})$

Knowing the stations of D and C, the stations of E and S are readily found and also the stations of the *C.S.* and *S.C.* by applying q_1 and q_2.

202. Problem. *Given I and R_c for circular curve* GHE, *also p and corresponding q for a spiral to fit the given curve.*



Required the distance BH $= h$ *through which the circular curve* GHE *must be moved in along* VO *to allow the use of this spiral; also the distance* GA $= d$ *from P.C. to T.S.*

$$\text{BH} = \text{PO} = \text{KG} = \frac{\text{KL}}{\cos \text{LKG}}$$

$$h = \frac{p}{\cos \frac{1}{2} I} \qquad (150)$$

$$\text{GA} = \text{AL} + \text{LG}$$

$$= \text{AL} + \text{LK} \tan \text{LKG}$$

$$d = q + p \tan \tfrac{1}{2} I \qquad (150\,A)$$

Problem. *Given I, R_c and h.*

Required p and d.

$$p = h \cos \tfrac{1}{2} I$$

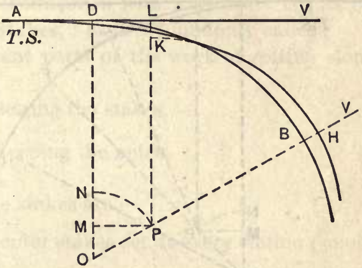
q is found by Table VII B or by § **193**.

$$d = q + p \tan \tfrac{1}{2} I.$$

In re-running old lines to introduce spirals, where an original circular curve is to be replaced by a spiral and a circular curve of the same degree, it is clear that the circular curve must necessarily be set in towards the center from H by a certain amount h. Practical considerations may often fix the distance h by which the curve must be moved. The method of § **193** will be found of considerable value in revisions of line since it allows great flexibility in the selection of spirals.

203. It may sometimes seem more desirable to change the radius of the circular curve so as to keep the new alignment in such position as to show as little deviation as possible from the old alignment and at the same time keep the length of line as nearly as possible unchanged. This may be accomplished as indicated in the figure below, where the line is carried outwards at B and inwards near D and L.

Problem. *Given I and R_1 for circular curve* DB; *also p of spiral; also* BH $=h$ *measured along* VO *locating* H *through which new circular curve is to pass to allow the use of this spiral.*



Required the radius $R_2 =$ KP *of the new curve* KH; *also q consistent with p and R_2; also distance* DA $= d$ *from P.C. to T.S.*

$$\mathrm{PO} = \mathrm{NO} = \mathrm{OB} + \mathrm{BH} - \mathrm{PH}$$

$$= R_1 + h - R_2$$

$$\mathrm{OM} = \mathrm{DO} - \mathrm{DM}$$

$$= \mathrm{DO} - \mathrm{PK} - \mathrm{KL}$$

$$= R_1 - R_2 - p$$

$$\mathrm{NM} = \mathrm{NO} - \mathrm{OM} = h + p$$

$$\mathrm{PO} \text{ vers } \mathrm{NOP} = \mathrm{NM}$$

$$(R_1 - R_2 + h) \text{ vers } \tfrac{1}{2} I = h + p$$

$$R_1 - R_2 + h = \frac{h + p}{\text{vers } \frac{1}{2} I} \qquad (151)$$

Find q from p and R_2, by § **193.**

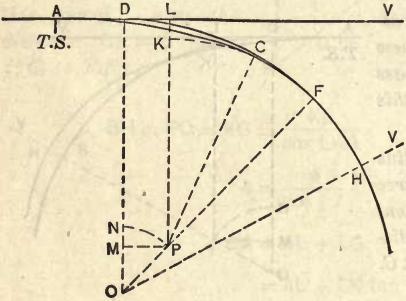
Then
$$\mathrm{DA} = \mathrm{AL} - \mathrm{DL}$$

$$= \mathrm{AL} - \mathrm{MP}$$

$$d = q - (R_1 - R_2 + h) \sin \tfrac{1}{2} I. \qquad (151\,A)$$

204. When it is necessary to keep the middle point H unchanged, on account of a bridge, or heavy embankment, or otherwise, it then becomes necessary to make part of the curve sharper, as CF in the figure below. The most practical method appears to be to assume the angle FOH, the part of the curve to remain unchanged; also assume the value of p and compute all other necessary data.

Problem. *Given I and R_1 of circular curve, also p of proposed spiral, also angle FOH $= I_1$ of the circular curve which is to remain unchanged.*



Required the radius R_2 of new curve CF, *to compound with original curve* FH; *also q consistent with p and R_2; also the distance* DA $= d$ *from P.C. to T.S.*

$$\text{FOH} = I_1$$

$$\text{DOH} = \tfrac{1}{2} I$$

$$\text{OP vers NOP} = \text{NM} = \text{MD} - \text{ND}$$

$$= \text{LP} - \text{KP} = p$$

$$(R_1 - R_2) \text{ vers } (\tfrac{1}{2} I - I_1) = p$$

$$R_1 - R_2 = \frac{p}{\text{vers } (\tfrac{1}{2} I - I_1)} \tag{152}$$

Find q from p and R_2 by § **193**.

Then

$$\text{DA} = \text{AL} - \text{DL}$$

$$= \text{AL} - \text{MP}$$

$$d = q - (R_1 - R_2) \sin (\tfrac{1}{2} I - I_1) \tag{152 A}$$

By making FOH $= I_1 = 0$, R_2 becomes continuous from the first spiral, through H and to its connection with the second spiral.

Another practical method would be to assume R_2 and p and compute I_1, q, d.

CHAPTER XI.

SETTING STAKES FOR EARTHWORK.

205. The first step in connection with Earthwork is staking out, or "**Setting Slope Stakes**," as it is commonly called.

There are two important parts of the work of setting slope stakes:

I. Setting the stakes.

II. Keeping the notes.

The data for setting the stakes are:

(*a*) The ground with center stakes set at every station (sometimes oftener).

(*b*) A record of bench marks, and of elevations and rates of grades established.

(*c*) The base and side slopes of the cross-section for each class of material.

In practice, notes of alignment, a full profile, and various convenient data are commonly given in addition to the above.

206. **I. Setting the Stakes.** The work consists of:

(*a*) Marking upon the back of the center stakes the "cut" or "fill" in feet and tenths, as

C 2.3 or F 4.7.

(*b*) Setting side stakes or slope stakes at each side of the center line at the point where the side slope intersects the surface of the ground, and marking upon the inner side of the stake the "cut" or "fill" at that point.

207. (*a*) The process of finding the cut or fill at the center stake is as follows:

Given for any station the height of instrument $= h_i$, *and the elevation of grade* $= h_g$.

Then the required *rod reading for grade*

$$r_g = h_i - h_g. \tag{153}$$

It is not necessary to figure h_g for each station.

Let $$h_{g_0} = h_g \text{ at Sta. } 0$$

$$h_{g_1} = h_g \text{ " " } 1$$

$$h_{g_2} = h_g \text{ " " } 2, \text{ etc.}$$

Also use similar notation for r_g.

Let $g =$ rate of grade (rise per station)

Then $$h_{g_1} = h_{g_0} + g$$

$$h_{g_2} = h_{g_1} + g$$

$$h_{g_3} = h_{g_2} + g, \text{ etc.}$$

$$r_{g_0} = h_i - h_{g_0}$$

$$r_{g_1} = h_i - h_{g_1}$$

$$= h_i - (h_{g_0} + g) = h_i - h_{g_0} - g$$

$$r_{g_1} = r_{g_0} - g \tag{154}$$

Similarly, $r_{g_2} = r_{g_1} - g$, etc.

It will be necessary, or certainly desirable, to figure h_g and r_g anew for each new h_i. It is well to figure h_g and r_g (as a check) for the last station before each turning point.

Example. $h_i = 106.25$

Sta. 0, grade elevation 100.00
5, " " 105.00 rate $+1.00$
10, " " 107.50 " $+0.50$

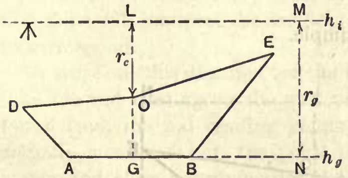
$$
\begin{array}{lllr}
r_{g_0} = 106.25 - 100.00 = 6.25 & & & 6.25 \\
r_{g_1} = & & 6.25 - \left.\begin{matrix} g \\ 1.00 \end{matrix}\right\} & = 5.25 \\
r_{g_2} = & & 5.25 - 1.00 & = 4.25 \\
r_{g_3} = & & 4.25 - 1.00 & = 3.25 \\
r_{g_4} = & & 3.25 - 1.00 & = 2.25 \\
r_{g_5} = & & 2.25 - 1.00 & = 1.25 \\
 & \text{Change in rate} & & \\
r_{g_6} = & & 1.25 - 0.50 & = 0.75 \\
r_{g_7} = & & 0.75 - 0.50 & = 0.25
\end{array}
$$

It is found necessary to take a *T.P.* here, and we therefore find

$$
\begin{aligned}
h_{g_7} &= h_{g_5} + 2g \\
&= 105.00 + 1.00 = 106.00 \\
r_{g_7} = h_i - h_{g_7} &= 106.25 - 106.00 = 0.25
\end{aligned}
$$

Therefore all intermediate values r_{g_1}, r_{g_2}, etc., are "checked."

208. Having thus found r_g, next, by holding the rod upon the surface of the ground at the center stake, the rod reading $r_c = \mathrm{LO}$ is observed from the instrument. The cut or fill



$$
\begin{aligned}
c = \mathrm{OG} &= \mathrm{MN} - \mathrm{LO} \\
&= r_g - r_c \qquad (155)
\end{aligned}
$$

In the figure given the values of r_g and c are positive; a positive value of c indicates a "cut," a negative value of c indicates a "fill."

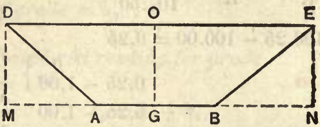
It can be shown that in the two cases of "fill,"

(1) When h_i is greater than h_g, and
(2) When h_i is less than h_g,

the formula given will hold good by paying due attention to the sign of r_g, whether $+$ or $-$.

209. (*b*) **Setting the Stake for the Side Slope.**

(1) *When the surface is level.*



Let $b = \text{AB} = $ base of section

$c = \text{OG} = $ center height

$s = \dfrac{\text{BN}}{\text{EN}} = \dfrac{\text{AM}}{\text{DM}} = $ side slope

$d = \text{OD} = \text{OE} = $ distance out

Then $d = \text{GB} + \text{BN}$

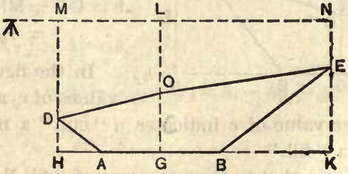
$= \frac{1}{2}b + s \times \text{DM} = \frac{1}{2}b + s \times \text{EN}$

$$= \tfrac{1}{2}b + sc \qquad (156)$$

Setting the Stake for the Side Slope.

(2) *When the surface is not level.*

Here the process is less simple.



Let $b = \text{AB} = $ base

$c = \text{OG} = $ center height (or cut)

$s = $ slope

$h_r = \mathrm{EK}$ = side height right

$h_l = \mathrm{DH}$ = " " left

$d_r = \mathrm{GK}$ = distance out right

$d_l = \mathrm{GH}$ = " " left

Then
$$\left.\begin{array}{l} d_r = \frac{1}{2}b + sh_r \\ d_l = \frac{1}{2}b + sh_l \end{array}\right\} \qquad (157)$$

But h_r and h_l are not known. It is evident from the figure that $h_r > c$ and $h_l < c$ in the case indicated, and therefore

$$d_r > \tfrac{1}{2}b + sc$$

$$d_l < \tfrac{1}{2}b + sc$$

It would be quite possible in many cases to take measurements such that the rate of slope of the lines OE and OD would be known, and the positions of E and D determined by calculation from such data. But speed and results finally correct are the essentials in this work, and these are best secured by finding h_l and h_r and the corresponding d_l and d_r upon the ground by a series of approximations, as described below.

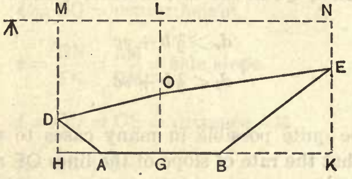
Having determined c, use this as a basis, and make an estimate at once as to the probable value of h_r at the point where the side slope will intersect the surface, and calculate $d_r = \frac{1}{2}b + sh_r$ to correspond.

Measure out this distance, set the rod at the point thus found, take the rod reading on the surface, and if the cut or fill thus found from the rod reading yields a value of d_r equal to that actually measured out, the point is correct. Otherwise make a new and close approximation from the better data just obtained, always starting with h_r and calculating d_r, and repeat the process until a point is reached where the cut or fill found from the rod reading yields a distance out equal to that taken on the ground. Then set the stake, and mark the cut or fill corresponding to h_r upon the inner side, as previously stated

Perform the same operation in a similar way to determine $d_l = \frac{1}{2}b + sh_l$, and mark this stake also upon the inner side with a cut or fill equal to h_l.

It requires a certain amount of work in the field to appreciate fully the process here outlined, but which in practice is very simple. It may impress some as being unscientific, and at first trial as slow, but with a little practice it is surprising how rapidly, almost by instinct, the proper point is reached, often within the required limits of precision at the first trial, while more than two trials will seldom be necessary, except in difficult country.

The instrumental work is just the same in principle as at the center stake.



Let $\quad r_r = \mathsf{NE} =$ rod reading at slope stake right,

then $\quad \mathsf{KN} - \mathsf{NE} = r_g - r_r = h_r$

here r_g is the same for center, right and left of section.

In some cases it may be necessary to make one or more resettings of the level in order to reach the side stakes from the center stake. In this case, of course, a new r_g must be calculated from the new h_i. This introduces no new principle, but makes the work slower.

A "slope-board" or "level-board" has quite frequently been used to advantage. In certain sections of country this might be considered almost indispensable. It consists simply of a long, straight-edge of wood (perhaps 15 ft. long) with a level mounted in the upper side. It is used with any self-reading rod. A rod quickly hand marked will serve the purpose well. Having given the cut or fill at the center, or at any point in the section, the leveling for the side stakes, and for any additional points, can readily, and with sufficient accuracy, be done by this "level-board," and the necessity for taking new turning points and resetting the level avoided.

210. II. *Keeping the Notes.*

The form of note-book used for keeping the notes of slope stakes and of center cuts and fills, often called "cross-section" notes, is shown on the following two pages.

The left-hand column for stations should read from bottom to top.

The surface elevations in column 2 are *not obtained directly from the levels*, but result from adding to the grade elevation at any station the cut or fill at that station, paying due attention to the signs. This column of surface elevations need not be entered up in the field, but may be filled in as office work more economically.

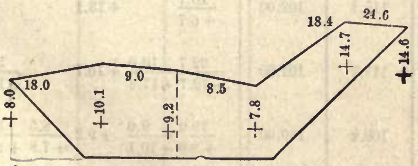
The column of grade elevations consists of the grade elevations as figured for each station.

The figures marked + are cuts in feet and tenths, and those marked − are fills; the figures above the cuts and fills are the distances out from the center, and the position in the notes, whether right or left of the center, corresponds to that on the ground.

The columns on the right-hand page are used for entering, when computed, the "quantities," or number of cubic yards, in each section of earthwork.

The column "**General Notes**" is used for entering extra measurements (of ditches, etc.) not included in the regular cross-section notes; also notes of material "hauled"; classification of material and various other matters naturally classed under the head of "Remarks."

When the surface is irregular between the center and side stakes, additional rod readings and distances out are taken, and the results entered as shown for station 0 on p. 144, the section itself being as shown below in the sketch.



Station 0

211. Form of Cross-Section Book (left-hand page).

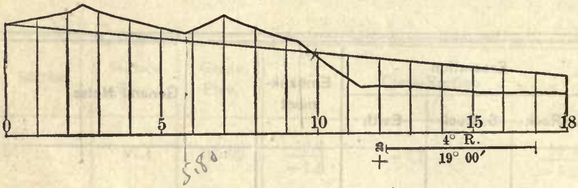
(Date)
(Names of Party)
Base 20; 1 to 1
14; $1\frac{1}{2}$ to 1

Station	Surface Elev.	Grade Elev.	Cross-Section					
5	97.1	105.00	$\frac{18.4}{-7.6}$		-7.9			$\frac{19.4}{-8.3}$
+69.7 *P.T.*	94.4	104.70	$\frac{22.1}{-10.1}$		-10.8			$\frac{23.0}{-10.7}$
4	96.9	104.00	$\frac{19.3}{-8.2}$		-7.1			$\frac{17.0}{-6.7}$
+27.2 *P.C.*	98.0	103.27	$\frac{16.6}{-6.4}$		-5.8			$\frac{12.4}{-3.6}$
3	98.1	103.00	$\frac{16.0}{-6.0}$		-4.9			$\frac{10.9}{-2.6}$
+87	100.6	102.87	$\frac{13.8}{-4.2}$		-2.3			$\frac{7.0}{0.0}$
+76	102.8	102.76	$\frac{10.3}{-2.2}$		0.0			$\frac{11.9}{+1.9}$
+64	103.7	102.64	$\frac{10.0}{0.0}$		$+1.1$			$\frac{13.2}{+3.2}$
+50	106.4	102.50	$\frac{13.4}{+3.4}$		$+3.9$			$\frac{17.1}{+7.1}$
2	115.1	102.00	$\frac{16.7}{+6.7}$		$+13.1$			$\frac{26.7}{+16.7}$
1	117.7	101.00	$\frac{22.7}{+12.7}$	$\frac{10.0}{+17.2}$	$+16.7$		$\frac{10.0}{+13.1}$	$\frac{22.2}{+12.2}$
0	109.2	100.00	$\frac{18.0}{+8.0}$	$\frac{9.0}{+10.1}$	$+9.2$	$\frac{8.5}{+7.8}$	$\frac{18.4}{+14.7}$	$\frac{24.6}{+14.6}$

212. **(Right-hand Page.)**

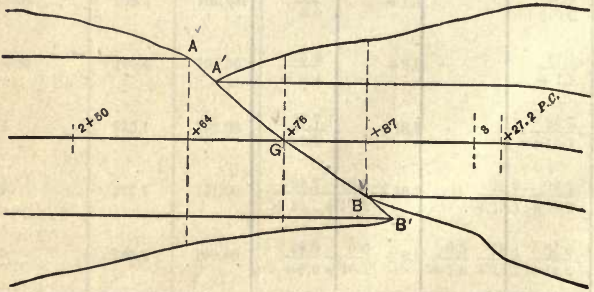
Excavation			Embankment	General Notes
L. Rock	S. Rock	Earth		

213. Cross-sections are taken at every full station, at every *P. C.* or *P. T.* of curve, wherever grade cuts the surface, and in addition, at every break in the surface. In the figure below, showing a profile, sections should be taken at the following stations: —



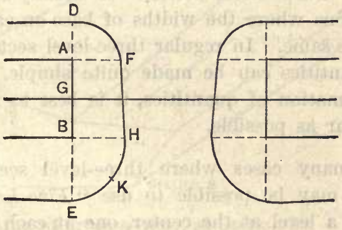
At Stations 0, 1, 2, 2 + 52, 3, 4, 5, 5 + 80, 6, 7, 8, 9, 9 + 29, 9 + 82, 10, 11, 11 + 30, 12, 12 + 25 *P. C.*, 13, 14, 15, 16, 17 *P. T.*, 18.

214. It is not necessary actually to drive stakes in *all* cases where a cross-section is taken and recorded, but in every case where they will aid materially in construction stakes should be set. It is best to err on the safe side, which is the liberal side. In passing from cut to fill, it is customary to take full cross-sections, not only at the point where the grade line cuts the surface at the *center* line of survey, but also where the grade cuts the surface at the outside of the base, both *right* and *left*, as in the figure below, which illustrates the notes on p. 144; full cross-sections are taken not only at stations 2 + 76, but also at 2 + 64 and 2 + 87.



215. Stakes are actually set at the center G and at the point A, where the outside line of the base of *Excavation* cuts the surface, and at B, where the outside line of the base of *Embankment* cuts the surface. It is not customary to set stakes or record the notes for the points A′ and B′. The stakes at A, G, and B are a sufficient guide for construction, and the solidities or "quantities" would in general be affected only slightly by the additional notes if they were made. When the line AGB crosses the center line nearly at right angles, it would not be necessary to take more than one section so far as the notes are concerned. It is well, however, to set the stakes A and B exactly in their proper position.

216. Wherever an opening is to be left in an Embankment for a bridge or for any other structure, stakes should be set as in the figure below: —



At A and B (at the side of the base and top of the slopes AF and BH) stakes should be set marked "*Bank to Grade*"; and at F and H (at the foot of the slopes) stakes should be set marked "*Toe of Slope.*" Where the bank is high, an additional stake K at foot of slope may be set as an aid to construction. The stakes at D and E should also be set as ordinary slope stakes.

217. The "level notes" proper, or the record of heights of instrument, bench marks, turning points, etc., used in setting slope stakes, are usually kept separate from the cross-section notes. One reason for this is that level notes run from top to bottom of page, while cross-section notes read from bottom to top of page. The level notes should be kept either in the back

of the cross-section book or in a level book carried for that purpose. Keeping these or any other notes on a slip of paper is bad practice.

218. Earthwork can be most readily computed when the section is a "*Level Section*," that is when the surface is level across the section; but this is seldom the case, and for purposes of final computation it is not often attempted to take measurements upon that basis.

219. In general, in railroad work, the ground is sufficiently regular to allow of "*Three-Level Sections*" being taken, one level (elevation) at the center and one at each slope stake, as shown by these notes, where Base is 20, and Slope $\frac{1}{2}$ to 1:—

$$\frac{11.3}{+2.6} \qquad +4.2 \qquad \frac{12.8}{+5.5}$$

The term "*Three-Level Section*" is usually applied only to *regular* sections where the widths of base on each side of the center are the same. In regular three-level sections the calculation of quantities can be made quite simple. To facilitate the final estimation of quantities, it is best to use three-level sections as far as possible.

220. In many cases where three-level sections are not sufficient, it may be possible to use "*Five-Level Sections*," consisting of a level at the center, one at each side where the *base* meets the side slope, and one at each side slope stake, as shown by the following notes:—

Base 20, Slope 1 to 1.

$$\frac{22.7}{+12.7} \qquad \frac{10.0}{+17.2} \qquad +16.7 \qquad \frac{10.0}{+13.1} \qquad \frac{22.2}{+12.2}$$

The term "*Five-Level Section*" is usually applied only to regular sections where the base and the side slopes are the same on each side of the center.

221. Where the ground is very rough, levels have to be taken wherever the ground requires, and the calculations must be made to suit the requirements of each special case, although certain systematic methods are generally applicable. Such sections are called "*Irregular Sections.*"

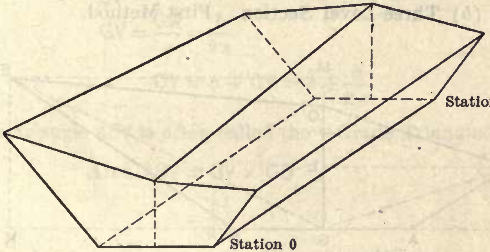
CHAPTER XII.

METHODS OF COMPUTING EARTHWORK.

222. In calculating the volumes or "quantities" of Earthwork, the principal methods used are as follows : —

I. Averaging End Areas. II. Prismoidal Formula.

223. I. Averaging End Areas.



Let A_0 = area of cross-section at Station 0

A_1 = " " " " " " 1

l = length of section, Sta. 0 to Sta. 1

V = volume of section of earthwork (Sta. 0 to 1)

Then $V = \frac{A_0 + A_1}{2} l$ (in cubic feet) (158)

$= \frac{A_0 + A_1}{2} \cdot \frac{l}{27}$ (in cubic yards) (159)

As (158) is capable of expression $V = A_0 \frac{l}{2} + A_1 \frac{l}{2}$

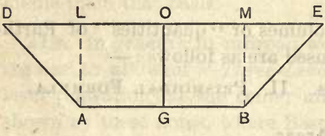
it is practically based on the assumption that the volume consists of two prisms, one of base A_0 and one of base A_1, and each of a length, or altitude of $\frac{l}{2}$.

224. To use this method, we must find the area A of each cross-section; the cross-section may be :—

(*a*) *Level.* (*b*) *Three-Level.* (*c*) *Five-Level.* (*d*) *Irregular.*

225. (*a*) **Level Cross-Section.**

Let $b = \text{base} = \mathsf{AB}$ $\quad s = \text{side slope} = \dfrac{\mathsf{DL}}{\mathsf{AL}} = \dfrac{\mathsf{EM}}{\mathsf{BM}}$



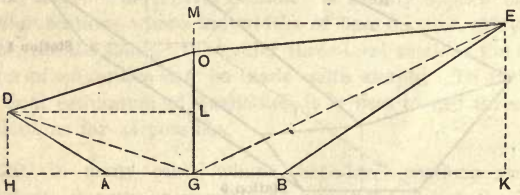
$c = \text{center ht.} = \mathsf{OG}$

$A = \text{area of cross-section}$

Then $\mathsf{DL} = \mathsf{EM} = sc$

$$A = \mathsf{AB} \times \mathsf{OG} + \mathsf{DL} \times \mathsf{AL}$$
$$= bc + sc^2$$
$$= c(b + sc) \qquad (160)$$

226. (*b*) **Three-Level Section.** First Method.



Let $b = \text{base} = \mathsf{AB}$ $\qquad s = \text{side slope}$

$c = \text{center height}$

$h_r = \text{side height } \mathsf{EK}$ $\qquad h_l = \text{side height } \mathsf{DH}$

$d_r = \text{distance out } \mathsf{ME}$ $\qquad d_l = \text{distance out } \mathsf{DL}$

$A = \text{area of cross-section}$

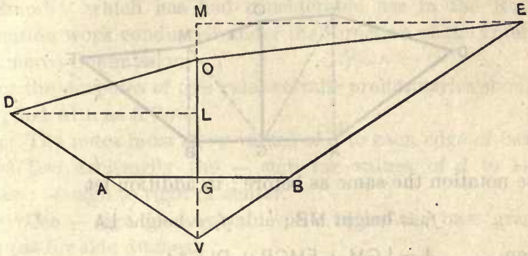
Then $A = \mathsf{OGD} + \mathsf{OGE} + \mathsf{GBE} + \mathsf{AGD}$

$$= \tfrac{1}{2}\mathsf{OG} \times \mathsf{DL} + \tfrac{1}{2}\mathsf{OG} \times \mathsf{ME} + \tfrac{1}{2}\mathsf{GB} \times \mathsf{EK} + \tfrac{1}{2}\mathsf{AG} \times \mathsf{DH}$$

$$= \tfrac{1}{2}c(d_l + d_r) + \tfrac{1}{2}\frac{b}{2}(h_r + h_l)$$

$$= \frac{c(d_l + d_r) + \dfrac{b}{2}(h_l + h_r)}{2} \qquad (161)$$

227. (*b*) **Three-Level Section.** Second Method.



Using the same notation.

$$\frac{\mathsf{GB}}{\mathsf{GV}} = s$$

$$\mathsf{GV} = \frac{\mathsf{GB}}{s} = \frac{b}{2\,s}$$

$$\mathsf{OV} = c + \mathsf{GV} = c + \frac{b}{2\,s}$$

The triangle ABV is often called the "Grade Triangle."

$$\text{Area ABV} = \mathsf{GV} \times \mathsf{GB}$$

$$= \frac{b^2}{4\,s}$$

$$\text{Area EODV} = \mathsf{OV} \times \frac{\mathsf{DL}}{2} + \mathsf{OV} \times \frac{\mathsf{ME}}{2}$$

$$= \left(c + \frac{b}{2\,s}\right)\frac{d_l + d_r}{2}$$

$$A = \mathsf{EODV} - \mathsf{ABV}$$

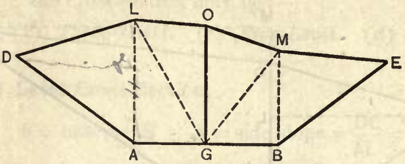
$$= \left(c + \frac{b}{2\,s}\right)\frac{d_l + d_r}{2} - \frac{b^2}{4\,s}$$

Let $D = d_l + d_r$

$$A = \left(c + \frac{b}{2\,s}\right)\frac{D}{2} - \frac{b^2}{4\,s} \qquad (162)$$

In using this formula for a *series* of cross-sections of the same base and slope, $\frac{b}{2\,s}$ and $\frac{b^2}{4\,s}$ are constants, and the computation of A becomes simple and more rapid than the first method

228. (*c*) **Five-Level Section.**



Use notation the same as before; in addition let

$$f_r = \text{height MB}; \qquad f_l = \text{height LA}$$

Then $A = \text{LGM} + \text{EMGB} + \text{DLGA}$

$$= \frac{cb}{2} + \frac{f_r d_r}{2} + \frac{f_l d_l}{2}$$

$$A = \frac{cb + f_r d_r + f_l d_l}{2} \qquad (163)$$

229. (*d*) **Irregular Section.**

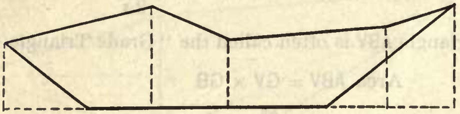


FIG. 1.

The "Irregular Section," as shown in the figure, may be divided into trapezoids by vertical lines, as in Fig. 1; or into triangles by vertical and diagonal lines, as in Fig. 2.

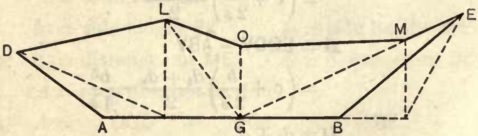


FIG. 2.

The triangles in Fig. 2 can be computed in groups of two, and with the advantage of one less numerical computation than is necessary in Fig. 1, proceeding as follows:

In Fig. 2, let h_D, d_D, etc. apply to points indicated by subscripts, and let OG be at the center line.

$$A = \tfrac{1}{2}[h_D(d_A - d_L) + h_L(d_D - 0) + h_O(d_L + d_M) + h_M(d_E - 0) - h_E(d_M - d_B)]$$

230. The principle involved has found expression in a "rule of thumb" which has had considerable use in the Railroad Valuation work conducted under the direction of the Interstate Commerce Commission.

For the purposes of this rule, certain preliminaries should be complied with as follows:

(*a*) The notes must show values of d to each edge of base.

(*b*) Use arbitrarily the $-$ sign for values of d to left of center; $+$ sign to right of center.

(*c*) Use $-$ sign for any value of h below the base grade in cuts (as for side ditches).

(*d*) Notes for points on original surface of ground should appear in brackets.

The rule is:

1. Start at any point; use every value of h in order, proceeding clockwise around the figure.

2. Multiply each value of h by $(d_a - d_b)$ using algebraic values. (Here d_a represents the value of d for point next in advance, and d_b for point next back.)

3. Find the algebraic sum of these and divide by 2.

The result is the area of the section.

The necessity for using values of h or d algebraically is confined to the purposes of this rule. Such values are not used algebraically in other parts of this book.

It is evident that the "rule of thumb" described applies correctly to the solution of the section shown in Fig. 2.

The rule may be shown to apply to Triangular Sections, and to Two Level Sections in which the center height is lacking.

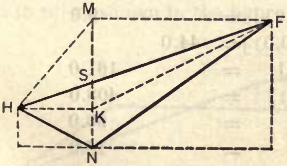


FIG. 3.

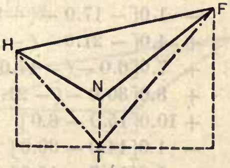


FIG. 4.

In Figures 3 and 4, N need not be on center line. In Fig. 3, draw lines, vertical NM, and horizontal HK and FM. Join HM and FK.

It may readily be shown that areas MSH = FSK.

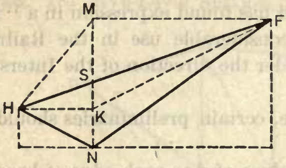


FIG. 3.

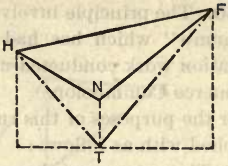


FIG. 4.

Then area $\quad \mathsf{HNF} = \mathsf{FNK} + \mathsf{HNM}$

$$= h_{\mathsf{H}}(d_{\mathsf{F}} - d_{\mathsf{N}}) + h_{\mathsf{F}}(d_{\mathsf{N}} - d_{\mathsf{H}})$$

which evidently complies with the rule when all values of d are to the right of center. It will apply equally in other cases by using values of d algebraically.

An example will more fully illustrate the use of this rule

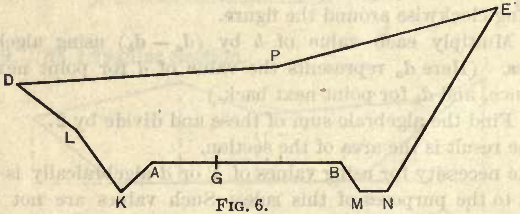


FIG. 6.

Notes: $\left(\dfrac{21.0}{+7.0}\right) \qquad \left(\dfrac{6.0}{+8.0}\right) \qquad \left(\dfrac{30.0}{+10.0}\right)$

$$\frac{17.0}{+4.0} \quad \frac{10.0}{-1.0} \quad \frac{8.0}{0.0} \quad \frac{0.0}{0.0} \quad \frac{12.0}{0.0} \quad \frac{14.0}{-1.0} \quad \frac{15.0}{-1.0}$$

		−	+
$-\ 1.0[-17.0-(-8.0)]$	=		9.0
$+\ 4.0[-21.0-(-10.0)]$	=	44.0	
$+\ 7.0[6.0-(-17.0)]$	=		161.0
$+\ 8.0[30.0-(-21.0)]$	=		408.0
$+\ 10.0[15.0-6.0]$	=		90.0
$-\ 1.0[14.0-30.0]$	=		16.0
$-\ 1.0[12.0-15.0]$	=		3.0

$$-44 + 687.0 = 643.0$$

$$2\underline{)643.0}$$

$$321.5 \text{ sq. ft.}$$

That the rule applies to this case may be shown by dividing Fig. 6 into pairs of triangles as is done in Fig. 2.

In Fig. 4, the rule applies to HTF. Subtracting the area of the pair of triangles HTFN, it then applies to triangle HNF.

In the Two Level Section, applying the rule to the triangle DGE, and combining with the areas DGA and EGB, evidently the rule still applies.

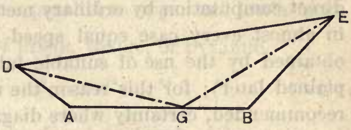
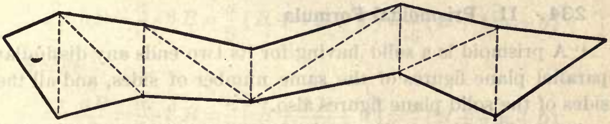


FIG. 5.

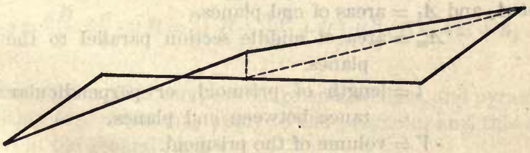
Other figures may be drawn, and the rule shown to apply by similarly dividing into pairs of triangles.

231. In the Sections shown in Figures 1, 2, 5, and 6, the base of a Section of Earthwork has been a plane. It may often happen that it will be necessary to take sections which are altogether irregular, with bases of varying widths and not in the form of a plane surface. The above rule will still hold good.

Where it is possible to take the elevations at the same points on the original surface and again on the excavated surface, as in the figure below, the computations can be simplified by dividing into triangles as shown in this figure.



A common form of section is one where part is in cut and part in fill as shown in the figure below.



This section also may be considered a special case of Irregular Section, and divided into convenient triangles, and into pairs of triangles so far as is feasible.

232. Another method which has been used for calculating irregular cross-sections is to plat them on cross-section paper, and get the area by "*Planimeter.*" In very irregular cross-sections this method would prove economical as compared with direct computation by ordinary methods, but it is probable that in almost every case equal speed and equal precision can be obtained by the use of suitable tables or diagrams (to be explained later); for this reason the use of the planimeter is not recommended, certainly where diagrams are available.

233. Whatever may be the form of section, or whatever the method of computation, having found the values of A for each cross-section, the volume V is found for the End Area Method, by the formula above given.

$$V = \frac{A_0 + A_1}{2} \cdot \frac{l}{27} \text{ (in cu. yds.)} \qquad (159)$$

It is found that this formula is only approximately correct. Its simplicity and *substantial accuracy* in the majority of cases render it so valuable that it has become the formula in most common use. It gives results, in general, larger than the true solidity.

234. II. Prismoidal Formula.

"A prismoid is a solid having for its two ends any dissimilar parallel plane figures of the same number of sides, and all the sides of the solid plane figures also."

Any prismoid may be resolved into prisms, pyramids, and wedges, having as a common altitude the perpendicular distance between the two parallel end planes.

Let A_0 and A_1 = areas of end planes.

A_m = area of middle section parallel to the end planes.

l = length of prismoid, or perpendicular distance between end planes.

V = volume of the prismoid.

Then it may be shown that

$$V = (A_0 + 4\,A_m + A_1)\frac{l}{6}$$

235. Let $B =$ area of lower face, or base of a prism, wedge, or pyramid.

$b =$ area of upper face.

$m =$ middle area parallel to upper and lower faces.

$a =$ altitude of prism, wedge, or pyramid.

$s =$ solidity " " " " "

Then the area of the *upper face* b in terms of *lower base* B will be for

Prism	Wedge	Pyramid
$b = B$	$b = 0$	$b = 0$

and the *middle area* m will be for

Prism	Wedge	Pyramid
$m = B$	$m = \frac{B}{2}$	$m = \frac{B}{4}$

The solidity s will be for

Prism

$$s = aB = \frac{a}{6} \cdot 6B = \frac{a}{6}(B + 4B + B) = \frac{a}{6}(B + 4m + b)$$

Wedge

$$s = \frac{aB}{2} = \frac{a}{6} \cdot 3B = \frac{a}{6}\left(B + \frac{4B}{2} + 0\right) = \frac{a}{6}(B + 4m + b)$$

Pyramid

$$s = \frac{aB}{3} = \frac{a}{6} \cdot 2B = \frac{a}{6}\left(B + \frac{4B}{4} + 0\right) = \frac{a}{6}(B + 4m + b)$$

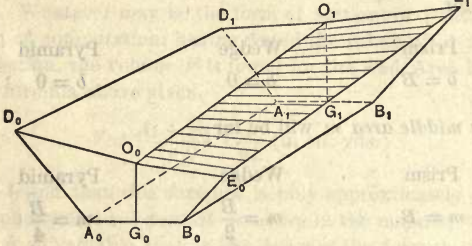
Since a prismoid is composed of prisms, wedges, and pyramids, the same expression may apply to the prismoid, and this may be put in the general form

$$V = (A_0 + 4A_m + A_1)\frac{l}{6} \tag{163 A}$$

using the notation of the preceding page.

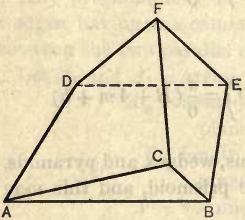
236. A regular section of earthwork having for its surface a plane face is a prismoid. Most sections of earthwork have not their surface plane, and are not strictly prismoids, although they are so regarded by some writers.

In this figure the lines E_0O_0 and E_1O_1 are not parallel, and therefore the surface $O_0O_1E_1E_0$ is not a plane. The most common assumption as to this surface is that the lines O_0O_1 and E_0E_1 are right lines, and that the surface $O_0O_1E_1E_0$ is a warped surface, generated by a right line moving as a generatrix always



parallel to the plane $O_0G_0B_0E_0$ and upon the lines O_0O_1 and E_0E_1 as directrices, as indicated in the figure. The surface thus generated is a warped surface called a "hyperbolic paraboloid." It will be shown that the "prismoidal formula" applies also to this solid, which is not, however, properly a prismoid.

237. In the case of a section whose ends are parallel and triangular in form, it may be shown that the prismoidal formula applies when one surface is plane whether the other two surfaces are plane or warped in the manner above outlined.



In the following figure, ABC and DEF are parallel, and the surfaces ACFD and BCFE may be considered warped surfaces, and ABED plane.

Let $b_0 =$ base AB ;

and $b_1 =$ DE

$h_0 =$ altitude of ABC ; and $h_1 =$ altitude of DEF

$l =$ distance between parallel end planes

$A_0 =$ area of ABC ; and $A_1 =$ area of DEF

Also use notation b_x, h_x, A_x for a section distant x from ABC.

Then $A_0 = \dfrac{b_0 h_0}{2} \qquad A_1 = \dfrac{b_1 h_1}{2}$

$$A_x = \frac{b_x h_x}{2} = \frac{1}{2}\left[b_0 - (b_0 - b_1)\frac{x}{l}\right]\left[h_0 + (h_1 - h_0)\frac{x}{l}\right]$$

$$V = \int_0^l \frac{1}{2}\left[b_0 + (b_1 - b_0)\frac{x}{l}\right]\left[h_0 + (h_1 - h_0)\frac{x}{l}\right]dx$$

$$= \frac{1}{2}\left\{ b_0 h_0 l + [b_0(h_1 - h_0) + h_0(b_1 - b_0)]\frac{l^2}{2\,l} + (b_1 - b_0)(h_1 - h_0)\frac{l^3}{3\,l^2}\right\}$$

$$= \frac{l}{12}\left\{\begin{array}{l} +\,6\,b_0h_0 + 3\,b_0h_1 + 3\,b_1h_0 + 2\,b_1h_1 \\ -\,3\,b_0h_0 - 2\,b_0h_1 - 2\,b_1h_0 \\ -\,3\,b_0h_0 \\ +\,2\,b_0h_0 \end{array}\right\}$$

$$V = \frac{l}{12}(2\,b_0h_0 + 2\,b_1h_1 + b_0h_1 + b_1h_0) \qquad (164)$$

238. Apply the "Prismoidal Formula" to the same section

$$V = \frac{l}{6}(A_0 + 4\,A_m + A_1)$$

$$= \frac{l}{6}\left[\frac{b_0h_0}{2} + 4\left(\frac{1}{2}\cdot\frac{b_0 + b_1}{2}\cdot\frac{h_0 + h_1}{2}\right) + \frac{b_1h_1}{2}\right]$$

$$= \frac{l}{12}(b_0h_0 + b_0h_0 + b_0h_1 + b_1h_0 + b_1h_1 + b_1h_1)$$

$$= \frac{l}{12}(2\,b_0h_0 + 2\,b_1h_1 + b_0h_1 + b_1h_0) \qquad (165)$$

This is identical in value and in form with the formula above (164). Therefore the "Prismoidal Formula" applies to the triangular section shown opposite.

The regular section of earthwork shown in the figure, page 161, is made up of triangular sections, to each of which the Prismoidal Formula applies, and so to the entire section.

Again it evidently will apply also to the prisms, wedges, and pyramids of which any "true prismoid" is composed.

239. In railroad earthwork the "Prismoidal Formula" is often burdensome in its application. For triangular sections and for regular "three-level" sections, the work may be simplified by computing the quantities first by the inexact method of "end areas," and then applying a *correction* which we may call the "**Prismoidal Correction.**"

Let V_e = solidity by end areas

V_p = solidity by prismoidal formula.

Then

$C = V_e - V_p$ = prismoidal correction.

In the triangular section

$$V_e = \frac{l}{2}\left(\tfrac{1}{2}\,b_0h_0 + \tfrac{1}{2}\,b_1h_1\right)$$

$$= \frac{l}{12}\left(3\,b_0h_0 + 3\,b_1h_1\right)$$

$$V_p = \frac{l}{12}\left(2\,b_0h_0 + 2\,b_1h_1 + b_0h_1 + b_1h_0\right)$$

$$C = V_e - V_p = \frac{l}{12}\left(b_0h_0 + b_1h_1 - b_0h_1 - b_1h_0\right)$$

$$= \frac{l}{12}(b_0 - b_1)(h_0 - h_1) \qquad (166)$$

which is the fundamental formula for prismoidal correction.

In the figure opposite, for the solid $O_0D_0G_0E_0E_1G_1D_1O_1$,

$$C = \frac{l}{12}\left[(c_1 - c_0)(d_{l_1} - d_{l_0}) + (c_1 - c_0)(d_{r_1} - d_{r_0})\right]$$

$$= \frac{l}{12}\ (c_1 - c_0)(d_{l_1} + d_{r_1} - d_{l_0} - d_{r_0})$$

Let $D_1 = d_{l_1} + d_{r_1}$ and $D_0 = d_{l_0} + d_{r_0}$

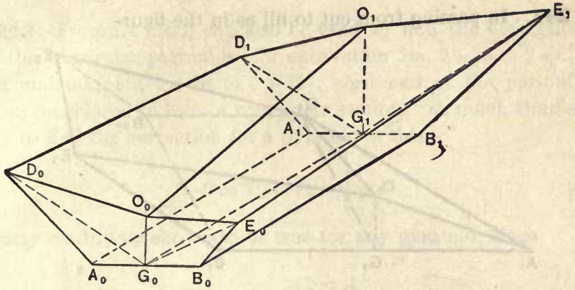
$$C = \frac{l}{12}\ (c_1 - c_0)(D_1 - D_0)$$

For the solid $G_0B_0E_0E_1B_1G_1$,

$$C = \frac{l}{12}\left(\frac{b_1}{2} - \frac{b_0}{2}\right)(h_{r_1} - h_{r_0}) = 0$$

Similarly for the solid $A_0G_0D_0D_1G_1A_1$.

So for the entire solid $C = \dfrac{l}{12}(c_1 - c_0)(D_1 - D_0)$ (167)



240. This formula can be used only when the width of base is the same at both ends of the section. From the method of its derivation it is evident that for the right half of a regular three level section

$$C_r = \frac{l}{12}(c_1 - c_0)(d_{r_1} - d_{r_0}) \qquad (167\ A)$$

When $l = 100$

$$C_{100} = \frac{100}{12 \times 27}(c_1 - c_0)(D_1 - D_0)$$

$$= \frac{1}{3.24}(c_1 - c_0)(D_1 - D_0) \text{ in cu. yds.} \qquad (168)$$

Since $C = V_e - V_p \qquad V_p = V_e - C \qquad (169)$

For a section of length l, $C_l = \frac{l}{100} C_{100}$

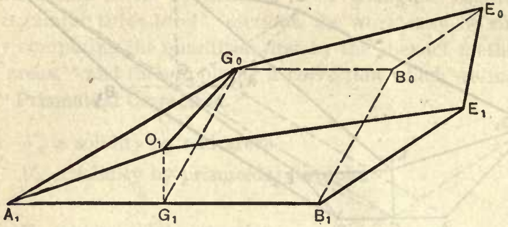
$$V_{pl} = \frac{l}{100}(V_{e100} - C_{100}) \qquad (169\ A)$$

For the purposes of prismoidal correction, it is simpler to use numerical values of c and D or d and neglect the sign + or −, since these are systematically used to represent cut or fill and the correction for any given numerical values of c and D is the same whether the section be cut or fill.

Therefore when $(c_1 - c_0)(D_1 - D_0)$ is *positive*, the arithmetical value of C is to be *subtracted* from V_e.

When $(c_1 - c_0)(D_1 - D_0)$ is *negative*, the arithmetical value of C is to be *added* to V_e. The latter case seldom occurs in practice, except where C is very small, perhaps small enough to be neglected.

241. In passing from cut to fill as in the figure

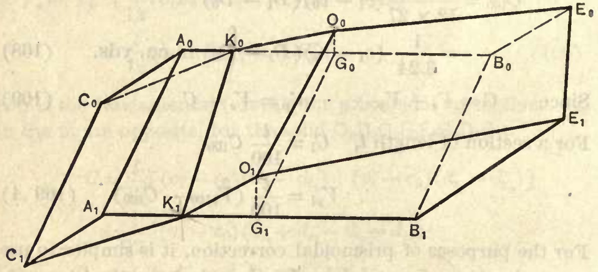


for the right half $\quad C_r = \dfrac{l}{12}(c_1 - c_0)(d_{r_1} - d_{r_0})$ from (167 *A*)

for the left side $\quad C_l = \dfrac{l}{12}(c_1 - c_0)\left(\dfrac{b}{2} - 0\right)$ from (166)

$$C = \frac{l}{12}(c_1 - c_0)(D_1 - d_{r_0})$$

For the special case of a side hill section



the prismoidal correction for cut will be

$$C_c = \frac{l}{12}(c_1 - c_0)(d_{r1} + d_{k1} - \overline{d_{r_0} + d_{k_0}})$$

the prismoidal correction for fill will be

$$C_f = \frac{l}{12}\left(h_{l_1} - h_{l_0}\right)\left(\frac{b}{2} - d_{k_1} - \overline{\frac{b}{2} - d_{k_0}}\right)$$

The quantities of cut and of fill will be kept separate, after applying the corrections.

242. Formula (166) can also be used to find the correction for the triangular pyramids (for excavation Sta. 2+76 to 2+87, and embankment 2 + 64 to 2 + 76), *each end of the pyramid* being considered to have a triangular section. A much simpler way to find the correction for a pyramid is this,

$$C = V_e - V_p = \frac{1}{3}V_e$$

as may readily be shown to be true for any pyramid, since

$$V_e = A\frac{l}{2}$$

$$V_p = A\frac{l}{3} = \frac{l}{3}\cdot\frac{1}{2}(w, h)\frac{1}{2}$$

$$C = V_e - V_p = A\frac{l}{6} \qquad (170)$$

since by the End Area method $V_e = A\frac{l}{2} + 0$

$$C = \frac{V_e}{3} \qquad (171)$$

243. In the case of regular "**Five-Level Sections**," as shown in the figure, p. 152, the prismoidal correction may be computed for each of the triangular masses bounded by

1. LGM 2. MEBG 3. LDAG

In the case of LGM, the prismoidal correction will evidently be $= 0$, since $D_0 = \text{AB} = D_1$, and therefore $D_0 - D_1 = 0$.

The correction for the mass bounded on one end by

$$\text{MEBG} = C = \frac{l}{12}(f_{r_0} - f_{r_1})(d_{r_0} - d_{r_1}) \qquad \text{from (166)}$$

and by $$\text{LDAG} = C = \frac{l}{12}(f_{l_0} - f_{l_1})(d_{l_0} - d_{l_1}) \qquad \text{from (166)}$$

244. In the case of "**Irregular Sections**," the prismoidal correction cannot with convenience be accurately employed. There are, however, several methods by which we may calculate a "prismoidal correction" which will be approximately correct, and good enough for practical purposes.

Inspection of the formula $C = \frac{l}{12}(c_1 - c_0)(D_1 - D_0)$ (167) makes it clear that the correction will be large when the two end sections differ much in size, and small when the end sections are nearly equal. Ordinarily in a large section both c and D are large. For any *given area of section* in a regular three-level section, if c is made smaller, D must be increased in nearly like measure, and formula (167) will show little change in the value of C even if c be changed, if the area remains the same.

For the purpose *only* of finding the prismoidal correction there are several approximate methods based on the principle above stated.

1. Where the section is only slightly irregular. Neglect all intermediate heights and figure correction from c and D. This is a very simple method.

Where more careful results seem desirable,

2. Find c and D for an "equivalent level section"; that is, a level section of equal area to the irregular section. Use the c and D thus determined in computing the prismoidal correction. These can be used with the c and D of a regular three-level section, or with the c and D of another equivalent level section.

The c and D of the equivalent level section may be found from Tables or from Diagrams, whose use will be shown in later chapters.

3. Find an equivalent regular three-level section (not level) either by

(*a*) retaining c and computing D, or

(*b*) retaining D and computing c.

The method of doing this will be made simple by Diagrams described in a later chapter.

4. Plot the irregular section on cross-section paper, and draw lines to form a regular three-level section which will closely approximate, in form, to the irregular section, and find c and D.

While the results obtained by any of the above methods are approximate, the resulting error can be only a small fraction of the entire correction, which is itself small.

The method of averaging end areas and applying the prismoidal correction allows of great rapidity, and secures great precision, and well meets the requirements of modern railroad practice.

CHAPTER XIII.

SPECIAL PROBLEMS.

245. Correction for Curvature.

In the case of a curve, the ends of a section of earthwork are not parallel, but are in each case normal to the curve. In calculating the solidity of a section of earthwork, we have heretofore assumed the ends parallel, and for curves this is equivalent to taking them perpendicular to the chord of the curve between the two stations.

Then, as shown in Fig. 1 (where IG and GT are center-line chords), the solidity (as above) of the sections IG and GT will be too great by the wedge-shaped mass RGP, and too small by

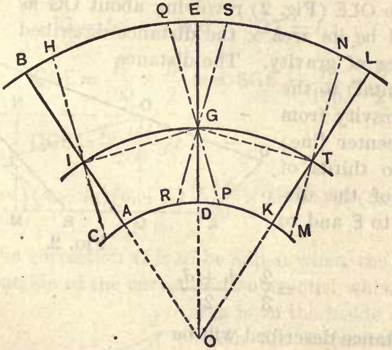


FIG. 1.

QGS. When the cross-sections on each side of the center are equal, these masses balance each other. When the cross-section on one side differs much in area from that on the other, the correction necessary may be considerable.

In Fig. 2, use c, h_l, h_r, d_l, d_r, b, s, as before.

Let D = degree of curve. Make BL = AD, and join OL.

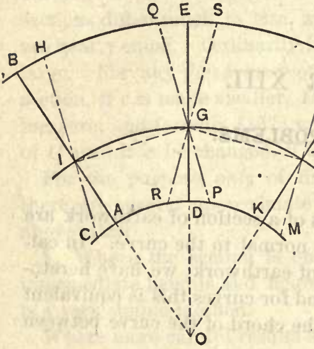


FIG. 1.

Then ODAG balances OLBG, and there remains an unbalanced area OLE.

Draw OKP parallel to AB.

By the "Theorem of Pappus" (see Lanza, Applied Mechanics), "If a plane area lying wholly on the same side of a straight line in its own plane revolves about that line, and thereby generates a solid of revolution, the volume of the solid thus generated is equal to the product of the revolving area and of the path described by the center of gravity of the plane area during the revolution."

The correction for curvature, or the solidity, developed by this triangle OLE (Fig. 2) revolving about OG as an axis will be its area × the distance described by its center of gravity. The distance out (horizontal) to the center of gravity from the axis (center line) will be two thirds of the mean of the distances out to E and to L, or

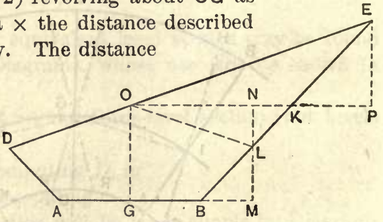


FIG. 2.

$$= \frac{2}{3} \cdot \frac{d_l + d_r}{2}$$

and the distance described will be

$$\frac{2}{3} \cdot \frac{d_l + d_r}{2} \times \text{angle QGS}$$

The area $\quad \text{OLE} = \text{OK} \times \frac{\text{NL} + \text{PE}}{2}$

$$= \left(\frac{b}{2} + sc\right) \frac{h_r - h_l}{2}$$

Therefore the correction for curvature,

$$C = \left(\frac{b}{2} + sc\right) \cdot \frac{h_r - h_l}{2} \cdot \frac{d_r + d_l}{3} \times \text{angle QGS}$$

When IG, GT are each a full station, or 100 ft. in length,

$$\text{QGS} = D$$

$$C = \left(\frac{b}{2} + sc\right) \cdot \frac{h_r - h_l}{2} \cdot \frac{d_r + d_l}{3} \times \text{angle } D$$

arc $1° = .01745$

$$C = \left(\frac{b}{2} + sc\right) \frac{h_r - h_l}{2} \times \frac{d_r + d_l}{3} \times 0.01745\, D$$

$$= \left(\frac{b}{2} + sc\right)(h_r - h_l)(d_r + d_l) \times 0.00291\, D \text{ (cu. ft.)} \qquad (172)$$

$$= \left(\frac{b}{2} + sc\right)(h_r - h_l)(d_r + d_l) \times 0.00011\, D \text{ (cu. yds.)} \qquad (173)$$

246. When IG or GT, or both, are less than 100 ft., let

$$\text{IG} = l_0 \qquad \text{and} \qquad \text{GT} = l_1$$

Then $\qquad \text{QGE} = \frac{l_0}{100} \times \frac{D}{2}$ and $\text{SGE} = \frac{l_1}{100} \times \frac{D}{2}$.

$$\text{QGS} = \frac{l_0 + l_1}{200} D$$

$$C' = \left(\frac{b}{2} + sc\right)(h_r - h_l)(d_r + d_l)\frac{l_0 + l_1}{200} \times 0.00011\, D \text{ (cu. yds.)} \qquad (174)$$

247. The correction C is to be added when the greater area is on the outside of the curve, and subtracted when the greater area is on the inside of the curve. When the center height is 0, as in Fig. 3, we may consider this a regular section in which $c = 0$, $h_l = 0$, and $d_l = \frac{b}{2}$; then

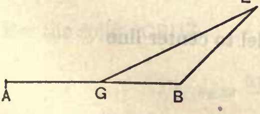


FIG. 3.

$$C = \frac{b}{2} \times h_r \times \left(d_r + \frac{b}{2}\right) \frac{l_0 + l_1}{200} \times 0.00011\, D \text{ (cu. yds.)} \qquad (175)$$

In the case of an irregular section, as shown in Fig. 4, the area and distance to center of gravity (for example, of OHEML) may be found by any method available, and the correction

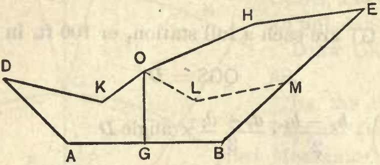
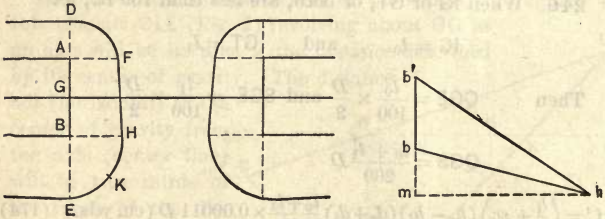


FIG. 4.

figured accordingly. The correction for curvature is, in present railroad practice, more frequently neglected than used. Nevertheless, its amount is sufficient in many cases to fully warrant its use.

248. Opening in Embankment.

Where an opening is left in an embankment, there remains outside the regular sections the mass DEKHF.



This must be calculated in 3 pieces, ADF, BEKH, ABHF.

Let b = base = AB

d_r = distance out right

d_l = distance out left

$$\left.\begin{array}{l} p_r = \text{BH} \\ p_l = \text{AF} \end{array}\right\} \text{taken parallel to center line}$$

$$\left.\begin{array}{l} f_r = \\ f_l = \end{array}\right\} \text{heights at} \left\{\begin{array}{l} \text{B} \\ \text{A} \end{array}\right.$$

s_1 = solidity ADF

s_2 = BEKH

s_3 = ABHF

Then (approximately) following the "Theorem of Pappus," s_1 = mean of triangular sections AD and AF × distance described by center of gravity.

In the quarter cone AFD, AF $= p_l$

$$\text{AD} = d_l - \frac{b}{2}$$

Then average radius $R_l = \text{mh} = \dfrac{\text{AF} + \text{AD}}{2}$

Area of vertical triangular section $A_l = \dfrac{f_l R_l}{2}$

Distance from A to center of gravity of vertical section $= \dfrac{R_l}{3}$

Arc described by center of gravity $= \dfrac{R_l}{3} \times \dfrac{\pi}{2} = \dfrac{\pi R_l}{6}$

$$s_1 = \frac{f_l R_l}{2} \times \frac{\pi R_l}{6} \text{ (cu. ft.)}$$

$$= \frac{f_l R_l^2 \times 3.1416}{2 \times 6 \times 27} \text{ (cu. yds.)}$$

$$s_1 = 0.0097\, f_l R_l^2 \text{ (cu. yds.)} \qquad (176)$$

Similarly, in the quarter cone BEKH

The average radius $R_r = \dfrac{\text{BH} + 2\,\text{BK} + \text{BE}}{4}$

$$s_2 = \frac{f_r R_r}{2} \times \frac{\pi R_r}{6} \text{ (cu. ft.)}$$

$$s_2 = 0.0097\, f_r R_r^2 \text{ (cu. yds.)} \qquad (177)$$

For the solid AGBHF

$$s_3 = \frac{\text{area AF} + \text{area BH}}{2} \times \text{AB}$$

$$= \frac{(f_l p_l + f_r p_r) b}{4} \qquad (178)$$

The work of deriving formulas (176) and (177) is approximate throughout, but the total quantities involved are in general not large, and the error resulting would be unimportant.

There seems to be no method of accurately computing this solidity which is adapted to general railroad practice.

249. Borrow-Pits.

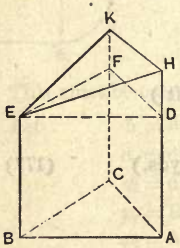
In addition to the ordinary work of excavation and embankment for railroads, earth is often "borrowed" from outside the limits of the work proper; and in such excavations called "borrow-pits," it is common to prepare the work by dividing the surface into squares, rectangles, or triangles, taking levels at every corner upon the original surface; again, after the excavation of the borrow-pit is completed, the points are reproduced and levels taken a second time. The excavation is thus divided into a series of vertical prisms having square, rectangular, or triangular cross-sections. These prisms are commonly truncated top and bottom. The lengths or altitudes of the vertical edges of these prisms are given by the difference in levels taken,

1st, on the original surface, and

2d, after the excavation is completed.

This method of measurement is very generally used, and for many purposes.

250. Truncated Triangular Prisms.



Let A = area of right section EFD of a truncated prism, the base ABC being a right section

h_1 = height AH

h_2 = " BE

h_3 = " CK

a = altitude of triangle EFD dropped from E to FD

Let V = volume of prism ABCKHE

s_l = solidity " " ABCFDE

s_u = " " pyramid FDEHK

Then $$s_l = A \times \mathrm{AD} = A \times \frac{3\,\mathrm{AD}}{3} = A \times \frac{\mathrm{AD} + \mathrm{BE} + \mathrm{CF}}{3}$$

$$s_u = \text{area DFKH} \times \frac{a}{3}$$

$$= \frac{\mathrm{KF} + \mathrm{HD}}{2} \times \mathrm{FD} \times \frac{a}{3}$$

$$= \frac{\mathrm{KF} + \mathrm{HD}}{3} \times \mathrm{FD} \times \frac{a}{2}$$

$$= \frac{\mathrm{KF} + \mathrm{HD}}{3} \times A$$

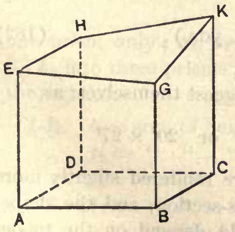
$$V = s_l + s_u = A\left(\frac{\mathrm{AD} + \mathrm{BE} + \mathrm{CF}}{3} + \frac{\mathrm{KF} + \mathrm{HD}}{3}\right)$$

$$= A\,\frac{(\mathrm{AD} + \mathrm{HD}) + \mathrm{BE} + (\mathrm{CF} + \mathrm{KF})}{3}$$

$$= A\,\frac{h_1 + h_2 + h_3}{3} \qquad (179)$$

If the prism be truncated top and bottom, the same reasoning holds and the same formula applies.

251. Truncated Rectangular Prism.



Let A = area of right section ABCD of a rectangular prism truncated on top (base is ABCD)

h_1 = height AE

h_2 = " BG

h_3 = " KC

h_4 = " HD

V = volume of prism

b = AD = BC

a = AB = DC

Then using method of end areas,

$$V = \frac{\mathsf{AEHD} + \mathsf{BGKC}}{2} \times a$$

$$= \frac{b\dfrac{h_1 + h_4}{2} + b\dfrac{h_2 + h_3}{2}}{2} \times a$$

$$= ab\,\frac{h_1 + h_2 + h_3 + h_4}{4}$$

$$V = A\,\frac{h_1 + h_2 + h_3 + h_4}{4} \text{ (cu. ft.)} \qquad (180)$$

$$V = \frac{A}{27} \cdot \frac{h_1 + h_2 + h_3 + h_4}{4} \text{ (cu. yds.)} \qquad (181)$$

We may find V, correct by the prismoidal formula, if we apply the prismoidal correction. The prismoidal correction $C = 0$, since $D_0 - D_1 = 0$ (or in this case $\mathsf{AD} - \mathsf{BC} = 0$). The formula therefore remains unchanged. It is evident from this, then, that the solution holds good, and the formula is correct, not only when the surface EHKG is a plane, but also when it is a warped surface generated by a right line moving always parallel to the plane ADHE, and upon EG and HK as directrices.

Some engineers prefer to cross-section in rectangles of base $15' \times 18'$. In this case

$$V = \frac{15' \times 18'}{27} \cdot \frac{h_1 + h_2 + h_3 + h_4}{4} \text{ (cu. yds.)}$$

$$= 10\,\frac{h_1 + h_2 + h_3 + h_4}{4} \text{ (cu. yds.)} \qquad (182)$$

Other convenient dimensions will suggest themselves, as

$$10' \times 13.5' \quad \text{or} \quad 20' \times 13.5' \quad \text{or} \quad 20' \times 27'$$

By this method the computations are rendered slightly more convenient; but the size of the cross-section, and the shape, whether square or rectangular, should depend on the topography. The first essential is accuracy in results, the second is simplicity and economy in field-work, and ease of computation should be subordinate to both of these considerations.

252. Assembled Prisms.

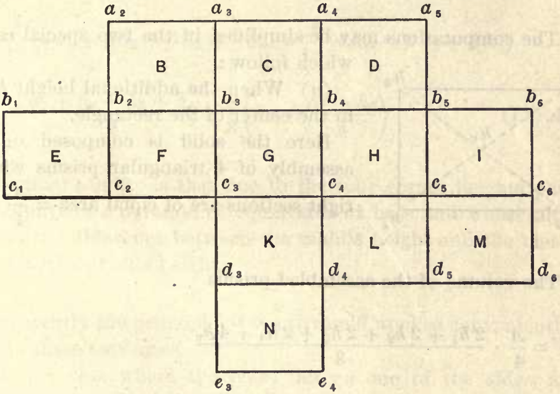
In the case of an assembly of prisms of equal base, it is not necessary to separately calculate each prism, but the solidity of a number of prisms may be calculated in one operation.

In the prism B,

$$V_B = A\frac{a_2 + a_3 + b_3 + b_2}{4}$$

$$V_C = A\frac{a_3 + a_4 + b_4 + b_3}{4}, \text{ etc.}$$

From inspection it will be seen, taking A as the common area of base of a single prism, and taking the sum of the solidities, that the heights a_2, a_5 enter into the calculation of



one prism only; a_3, a_4 into two prisms each; b_1, b_6 one only; b_2, b_5 into three prisms; b_3, b_4 into four prisms; and similarly throughout.

Let t_1 = sum of heights common to one prism
t_2 = " " " " " two prisms
t_3 = " " " " " three "
t_4 = " " " " " four "

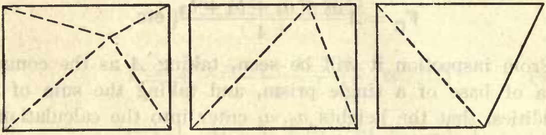
Then the total volume,

$$V_t = A\frac{t_1 + 2\,t_2 + 3\,t_3 + 4\,t_4}{4} \text{ (cu. ft.)} \qquad (183)$$

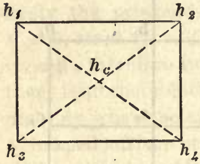
$$V_t = \frac{A}{27}\cdot\frac{t_1 + 2\,t_2 + 3\,t_3 + 4\,t_4}{4} \text{ (cu. yds.)} \qquad (184)$$

253. Additional Heights.

When the surface of the ground is rough it is not unusual to take additional heights, the use of which, in general, involves appreciable labor in computation, it being necessary commonly to divide the solid into triangular prisms, as suggested by the figures just below, which include the case of a trapezoid.



The computations may be simplified in the two special cases which follow:



(*a*) When the additional height h_c is in the center of the rectangle.

Here the solid is composed of an assembly of 4 triangular prisms whose right sections are of equal area $= \frac{A}{4}$.

The volume of the assembled prisms

$$V = \frac{A}{4} \cdot \frac{2h_1 + 2h_2 + 2h_3 + 2h_4 + 4h_c}{3}$$

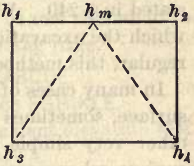
$$= \frac{A}{12}(2h_1 + 2h_2 + 2h_3 + 2h_4 + 4h_c)$$

$$= \frac{A}{12}(3h_1 + 3h_2 + 3h_3 + 3h_4) + \frac{A}{12}(4h_c - h_1 - h_2 - h_3 - h_4)$$

$$V = A\frac{h_1 + h_2 + h_3 + h_4}{4} + \frac{A}{3}\left(h_c - \frac{h_1 + h_2 + h_3 + h_4}{4}\right) \qquad (185)$$

or the total volume is that due to the four corner heights plus the volume of a pyramid of equal area of base and whose altitude is the difference between the center height and the mean of the four corner heights.

(*b*) When the additional height is at the middle of one side of the rectangle.



$$V = \frac{1}{3} \cdot \frac{A}{4} (h_1 + h_m + h_3 + h_2 + h_4 + h_m) +$$

$$\frac{1}{3} \cdot \frac{A}{2} (h_m + h_4 + h_3)$$

$$V = \frac{A}{12} (h_1 + h_m + h_3 + h_2 + h_4 + h_m + 2\,h_m + 2\,h_4 + 2\,h_3)$$

$$= \frac{A}{12} (h_1 + h_2 + 3\,h_3 + 3\,h_4 + 4\,h_m)$$

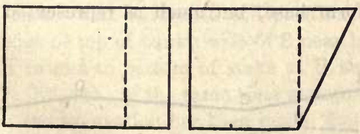
$$= \frac{A}{12} (3\,h_1 + 3\,h_2 + 3\,h_3 + 3\,h_4) + \frac{A}{12} (4\,h_m - 2\,h_1 - 2\,h_2)$$

$$= A\,\frac{h_1 + h_2 + h_3 + h_4}{4} + \frac{A}{3}\left(h_m - \frac{h_1 + h_2}{2}\right) \qquad (185\,A)$$

or the total solidity is that due to the four corner heights plus the solidity of a pyramid of equal area of base and whose altitude is the difference between the middle height and the mean of the adjacent side heights.

Apparently the principle of the pyramid applies conveniently only in these two cases.

For the case where the point lies on one of the sides, an alternate method of dividing the rectangle (or trapezoid) is indicated below.



The details of the computation in this case need not be worked out here.

254. The common practice in the case of borrow-pits is that stated in § 249. When the original surface and the surface to which the excavation is made are both somewhat rough and irregular, this method is naturally and properly adopted.

In many cases of excavation, the work is carried to a finished surface, sometimes a plane surface, or several planes, or some other very simple surface, sometimes to a more complicated surface where cross-sectioning the finished surface would not readily allow the facts to be shown on the plan.

In either of these cases the following method seems preferable.

(*a*) Cross-section the original surface as before.

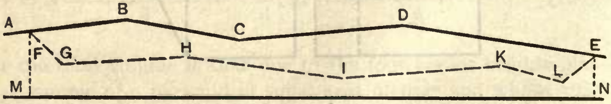
(*b*) Assume a convenient horizontal plane, slightly lower than the surface to which the excavation has been carried.

(*c*) Find the total earthwork to the original cross-sectioned surface, above this assumed plane as a base.

(*d*) Find the total earthwork to the finished surface, above the assumed plane as a base. In many cases this surface will be bounded by only a few planes and thus will allow very simple computations.

(*e*) Find the difference between (*c*) and (*d*); this will give the amount of earthwork excavated.

255. It often happens that an excavation is made of considerable length and not great breadth, and often of not great depth. In stripping soil under a proposed embankment these conditions prevail. The excavation can then best be handled very much as excavation is handled on railroads. A *line* will be run, and a series of cross-sections taken, the *line* serving as a center line, and cross-sections being taken at + stations along the line as often as required by the surface conditions. The cross-sections will be very irregular, not having any uniform base, but much as represented in the figure below.



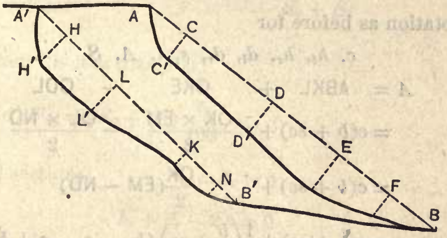
256. To find the area of these irregular sections, it may frequently happen that the best method may be one similar to that described for cross-sectioning on the preceding page.

(*a*) Find elevations on original surface ABCDE.
(*b*) Find elevations on excavated surface FGHIKLE.
(*c*) Assume a horizontal line at a convenient elevation MN.
(*d*) Calculate area MFBCDEN.
(*e*) Calculate area MFGHIKLEN.
(*f*) Area required is the difference between (*d*) and (*e*).

This method is simple in principle, and desirable in many cases. Where there are few sections to be computed it may be economical to use any method already well understood, rather than look up a method less familiar. Where many sections are to be computed, the "rule" of page 153 will prove economical.

257. It is frequently necessary to find the excavation made by digging into the side of a high bank. Cross-section points on a steep slope, often in loose sand, cannot be expected to yield good results for computing excavation.

In such cases the following method may prove valuable.



(*a*) Determine with care both the position and elevation of point A at edge of top of bank; also of B near bottom of slope.

(*b*) Sight from A to bottom of stake at B and read on leveling rod CC′, DD′, etc., at the same time measuring AC, AD, etc.

(*c*) After the excavation has been made, find the positions of A′ and B′; also the distances HH′, LL′, etc.; also A′H, A′L, etc.

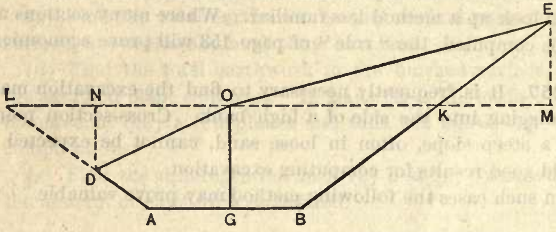
(*d*) Plot on cross-section paper and measure area between original surface and excavated surface. This can probably be done to best advantage by planimeter.

CHAPTER XIV.

EARTHWORK TABLES.

258. The calculation of quantities can be much facilitated by the use of suitably arranged "Earthwork Tables."

For regular "Three-Level Sections" very convenient tables can be calculated upon the following principles or formulas:—



Use notation as before for

$$c,\ h_l,\ h_r,\ d_l,\ d_r,\ s,\ l,\ A,\ S$$

Then
$$A = \text{ABKL} + \text{OKE} - \text{ODL}$$
$$= c(b + sc) + \frac{\text{OK} \times \text{EM}}{2} - \frac{\text{OL} \times \text{ND}}{2}$$
$$= c(b + sc) + \frac{\text{OK}}{2}(\text{EM} - \text{ND})$$
$$= c(b + sc) + \frac{1}{2}\left(\frac{b}{2} + sc\right)(h_r - c - c + h_l)$$
$$A = c(b + sc) + \frac{1}{2}\left(\frac{b}{2} + sc\right)(h_l + h_r - 2c)$$

For a prism of base A and $l = 50$, the solidity

$$S = 50\,A \text{ (cu. ft.)} = \frac{50}{27}\,A \text{ (cu. yds.)}$$

$$S = \frac{50}{27}c(b + sc) + \frac{25}{27}\left(\frac{b}{2} + sc\right)(h_l + h_r - 2c) \text{ (cu. yds.)} \qquad (186)$$

259. For cross-sections of a given base and slope, that is, given b and s constant, we may calculate for successive values of c, and tabulate values of L and K as follows: —

	L	K
c	$\frac{50}{27}c(b+sc)$	$\frac{25}{27}\left(\frac{b}{2}+sc\right)$

L represents the solidity for the *level section.*

K is for use as a correction. The formula then adapts itself to this table for any desired values of c, h_l, h_r.

$$S = L + K(h_l + h_r - 2c) \qquad (186\text{ A})$$

Having found for successive stations S_0 and S_1 (each for a prism $l = 50$), then for the *full section* by "end areas,"

$$V_{e100} = S_0 + S_1$$

for $$V_{e100} = \frac{A_0 + A_1}{2} \cdot \frac{100}{27} = \frac{50\,A_0}{27} + \frac{50\,A_1}{27}$$

$$V_{e100} = S_0 + S_1 \qquad (187)$$

260. When l is less than 100,

$$V_{el} = (S_0 + S_1)\,\frac{l}{100} \qquad (188)$$

For level sections $$h_l = h_r = c$$

$$h_l + h_r - 2c = 0$$

and the formula

$$S = L + K(h_l + h_r - 2c)$$

becomes $$S = L \qquad (189)$$

for level sections, and the quantities for any given values of c can be directly taken from column L without any correction from column K.

In preliminary estimates, or wherever center heights only are used, such tables are rapidly used.

261. Tables may be found in Allen's Tables XXXII for various bases for

$b = 20$	$s = 1\frac{1}{2}$ to 1	p. 252
$b = 14$	$s = 1\frac{1}{2}$ to 1	p. 248

An example will illustrate their use,

$$b = 14 \qquad s = 1\tfrac{1}{2} \text{ to } 1$$

Notes: —

Sta. 1	$\dfrac{16.0}{-6.0}$	-3.7	$\dfrac{12.4}{-3.6}$
0	$\dfrac{10.6}{-2.4}$	-2.5	$\dfrac{10.3}{-2.2}$

Calculations: —

3.7 $\quad L = 134.0 \quad K = 11.6 \quad h_l + h_r = 9.6$

$$\begin{array}{rrr} & & 2c = 7.4 \\ L = 134.0 & K = 11.6 & h_l + h_r = 9.6 \\ +\ \underline{25.5} & \underline{2.2} & 2c = \underline{7.4} \\ S_1 = 159.5 & 2.32 & +2.2 \\ & \underline{23.2} & \\ & 25.52 & \end{array}$$

2.5

$$\begin{array}{rrr} L = \ \ 82.2 & K = 10.0 & h_l + h_r = 4.6 \\ -\ \ \underline{4.0} & \underline{0.4} & 2c = \underline{5.0} \\ S_0 = \ \ 78.2 & 4.00 & -0.4 \end{array}$$

$$V_{100} = S_1 + S_0 = 237.7$$

262. There is also in Allen's Tables XXXI a "Table of Prismoidal Correction" calculated by the formula

$$C = \frac{1}{3.24}(c_0 - c_1)(D_0 - D_1)$$

In the example above

$$c_0 - c_1 = \ \ 2.5 - \ \ 3.7 = -1.2$$
$$D_0 - D_1 = 20.9 - 28.4 = -7.5$$

From Table find opp. 7.5 for 1 $\quad$ 2.31

$$\begin{array}{lll} & 2 & \qquad\qquad\qquad \underline{4.63}(10 \\ V_{100} = V_e = 237.7 & 0.2 & \quad \underline{0.46} \qquad\qquad 0.46 \\ \quad\ \ C = \underline{\ \ \ 2.8} & & C = 2.77 \\ \quad\ V_p = 234.9 & & \end{array}$$

263. When the section is less than 100 ft. in length, the prismoidal correction is made before multiplying by $\frac{l}{100}$

that is, $$V_{pl} = (S_0 + S_1 - C)\frac{l}{100} \qquad (190)$$

264. Equivalent Level Sections.

The Table of p. 179 (or Table XXXII, Allen's Tables) shows in the L column the value of $S = \frac{50}{27}A$ for values of center height c. Conversely if there be given the S of any section, "irregular" or "regular three level," the value of c for a level section of the same area may be found from the L column.

Example. From p. 180, Base 14, Slope $1\frac{1}{2}$ to 1 for

$S_1 = 159.5$ $\qquad c = 4.2 \qquad$ from Table XXXII

The notes of this section will be

$$\frac{13.3}{-4.2} \qquad -4.2 \qquad \frac{13.3}{-4.2}$$

265. For general calculations adapted both to regular "Three-Level Sections" and to "Irregular Sections," tables can be calculated upon the following principles and formulas:—

These tables are, in effect, tables of "Triangular Prisms," in which, having given (in feet) the base B and altitude a of any triangle, the tables give the solidity (in cubic yards) for a prism of length $l = 50$; that is,

$$S = \frac{aB}{2} \cdot \frac{50}{27} = \frac{50}{54} aB \qquad (191)$$

Whenever the calculations can be brought into the form $S = \frac{50}{54} aB$, the result can be taken directly from the table.

266. In Allen's Field and Office Tables, "Three-Level Sections" are provided for in Table XXXII for slope of $1\frac{1}{2}$ to 1 and bases 14 to 30. "Prismoidal Corrections" are found in Table XXXI; and "Triangular Prisms" in Table XXX.

267. In the tables the formula $S = \frac{50}{54} aB$ takes form thus, $S = \frac{50}{54} \times$ *width* $\times$ *height*, and the tables are arranged as below.

	Heights.
Widths	$\frac{50}{54}$ *width* $\times$ *height*

The application to "Three-Level Sections" is as follows: — We have formula (162), p. 151,

$$A = \left(c + \frac{b}{2s}\right)\frac{D}{2} - \frac{b^2}{4s}$$

and for a prism 50 ft. in length ($l = 50$)

$$S = \frac{50}{27} A = \frac{50}{54}\left(c + \frac{b}{2s}\right) D - \frac{50}{54} \cdot \frac{b}{2s} \cdot b \qquad (192)$$

or S is the sum of two quantities, each of which is in proper form for the use of the tables.

For cross-sections of a given base and slope (b and s constants), $\frac{b}{2s}$ is a constant, and also $\frac{50}{54} \cdot \frac{b}{2s} \cdot b$ is constant.

We may then calculate once for all $\frac{b}{2s}$, and call this B (a constant).

Also $\frac{50}{54} \cdot \frac{b}{2s} \cdot b$, and call this a constant E.

Then $$S = \frac{50}{54}(c + B)D - E$$

In using the tables, $c + B =$ height

$$D = \text{width}$$

As in the previous tables, having found S_0 and S_1,

$$V_{100} = S_0 + S_1$$

and $$V_l = (S_0 + S_1)\frac{l}{100}$$

268. Example. Allen's Tables XXX.

Notes :—

Sta. 1	$\dfrac{9.1}{-2.4}$	-1.2	$\dfrac{7.3}{-1.2}$
Sta. 0	$\dfrac{8.8}{-2.2}$	-0.7	$\dfrac{6.4}{-0.6}$

$b = 11$ $s = 1\frac{1}{2}$ to 1

$$\frac{b}{2s} = 3.7 = B$$

Grade triangle, $\frac{50}{54} \times 3.7 \times 11$

Under height 3.7, find	$1 = 3.43$	$10. = 34.3$
	$1 = 3.43$	$1. = 3.4$
		$E = 37.7$

Station 1. $c = 1.2$

$B = 3.7$

height $= 4.9$

$D = 9.1 + 7.3 = 16.4$

Under height 4.9, find	$1 = 4.54$	$10. = 45.4$
	$6 = 27.22$	$6. = 27.2$
	$4 = 18.15$	$.4 = 1.8$
		74.4
		$E = 37.7$
		$S_1 = 36.7$

Station 0. $c = 0.7$

$B = 3.7$

height $= 4.4$

$D = 8.8 + 6.4 = 15.2$

Under height 4.4, find	$1 = 4.07$	$10. = 40.7$
	$5 = 20.37$	$5. = 20.4$
	$2 = 8.15$	$.2 = 0.8$
		61.9
		$E = 37.7$
		$S_0 = 24.2$

$$V = S_1 + S_0 = 60.9$$

269. Irregular Sections

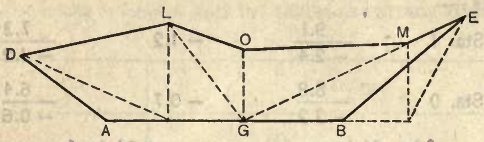


FIG. 2.

The scheme of computation should be the same as that used with pairs of triangles in Fig. 2, § 229, or as shown by the "rule" of page 153.

Instead of $\frac{1}{2}[h_D(d_A - d_L) + \text{etc.}]$ of Fig. 2, the Tables will give $\frac{50}{54}[h_D(d_A - d_L) + \text{etc.}]$.

So that the summation will give the result in cubic yards.

In a similar way the "Diagrams" to be described in the next Chapter will give $\frac{50}{54}[h_D(d_A - d_L)$ etc.].

Similar computations may be made by Slide Rule set in such a way that $\frac{50}{54}[h_D(d_A - d_L)$ etc.] will be taken off from the Slide Rule as the result of the computation.

If some computers prefer to plat cross-sections and compute by planimeter, the planimeter arm may be so adjusted as to record $\frac{50}{27} A$ rather than A.

Results by Diagram, by Slide Rule, or by Planimeter, will all be subject to the lack of precision involved in "readings." Any such lack of precision will be far less than the lack of precision due to determining the rod readings on the surface of the ground from which cuts and fills are computed, and therefore not objectionable.

CHAPTER XV.

EARTHWORK DIAGRAMS.

270. Computations of earthwork may also be made by means of diagrams from which results may be read by inspection merely.

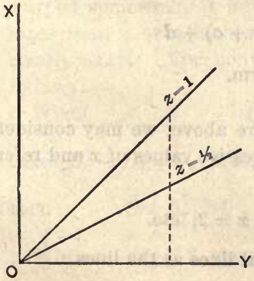
The principle of their construction is explained as follows: —

Given an equation containing three variable quantities as

$$x = zy \tag{194}$$

If we assume some value of z (making z a constant), the equation then becomes the equation of a right line.

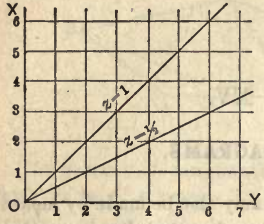
If this line be platted, using rectangular coördinates (as the line $z = 1$ in the figure), then having given any value of y, the corresponding value of x may be taken off by scale. If a new value of z be assumed, the equation is obtained of a new line which may also be platted (as $z = \frac{1}{2}$ in the figure), and from which also, having given any value of y, the corresponding value of x may be determined by scale. Assuming a series of values of z and platting, we have a series of lines, each representing a different value of z, and from any one of which, having given a value of y, we may by scale determine the value of x.



Thus, *given*, values of z and y; *required*, x, we may find,

1. The line corresponding to the given value of z, and
2. Upon this line we may find the value of x corresponding to the given value of y.

271. Next, if instead of platting upon *lines* as coördinate axes, we plat upon cross-section paper, the cross-section lines form a scale, so that the values of x and y need not be *scaled*, but may be *read* by simple inspection as in the figure.



272. If the equation be in the form

$$x = azy \qquad (195)$$

the same procedure is equally possible, and the line representing any value of z will still be a right line.

If the equation be in the form

$$x = a(z + b)(y + c) + d \qquad (196)$$

in which a, b, c, d, are constants, the same procedure is still possible, and the line representing a given value of z is a right line, as before.

The use of diagrams of this sort is therefore possible for the solution of equations in the form of

$$x = a(z + b)(y + c) + d$$

or in simpler modifications of this form.

273. Referring again to the figure above, we may consider the horizontal lines to represent successive values of x and refer to them as the lines

$$x = 0\,;\; x = 1\,;\; x = 2, \text{ etc.}$$

and similarly we may refer to vertical lines as the lines

$$y = 0\,;\; y = 1\,;\; y = 2, \text{ etc.}$$

just as we refer to the inclined lines

$$z = \tfrac{1}{2}\,;\; z = 1, \text{ etc.}$$

Having given any two of the quantities x, y, z, the third may be found by inspection from the diagram by a process similar to that described.

274. Diagram for Prismoidal Correction.

Formula $$C = \frac{1}{3.24}(c_0 - c_1)(D_0 - D_1) \qquad (168)$$

This has the form $x = \quad a \times \quad z \quad \times \quad y$

Construction of diagram.

Assume (as we did for z) a series of values of

$$c_0 - c_1 = 0,\ 1,\ 2,\ 3,\ 4,\ 5, \text{ etc.}$$

When $c_0 - c_1 = 0$ then $C = 0$
or, the line $c_0 - c_1$ coincides with the line $C = 0$.

When $c_0 - c_1 = 1$, the equation of the line $c_0 - c_1$ is

$$C = \frac{1}{3.24}(D_0 - D_1)$$

To piat this right line, we must find two or more points on the line. For the reason that cross-section paper is generally warped somewhat, it is best to take a number of points not more than 3 or 4 inches apart, in order to get the lines sufficiently exact. For convenience, take values of $D_0 - D_1$ as follows:—

When $(c_0 - c_1) = 1$

take $D_0 - D_1 = 0$, 3.24, 6.48, 9.72, 12.96, 16.20, etc.

then $C = 0$, 1 , 2 , 3 , 4 , 5 , etc.

When $c_0 - c_1 = 2$, the equation of the line $c_0 - c_1$ is

$$C = \frac{1}{3.24} \cdot 2(D_0 - D_1)$$

Therefore when $c_0 - c_1 = 2$

take $D_0 - D_1 = 0$, 3.24, 6.48, 9.72, 12.96, 16.20, etc.

then $C = 0$, 2 , 4 , 6 , 8 , 10 , etc.

275. In like manner a table may be constructed.

	0	3.24	6.48	9.72	12.96	16.20	19.44	22.68	25.92	$D_0 - D_1$
0	0	0	0	0	0	0	0	0	0	
1	0	1	2	3	4	5	6	7	8	
2	0	2	4	6	8	10	12	14	16	
3	0	3	6	9	12	15	18	21	24	
4	0	4	8	12	16	20	24	28	32	
5	0	5	10	15	20	25	30	35	40	
6	0	6	12	18	24	30	36	42	48	
7	0	7	14	21	28	35	42	49	56	
8	0	8	16	24	32	40	48	56	64	
9	0	9	18	27	36	45	54	63	72	
10	0	10	20	30	40	50	60	70	80	
$c_0 - c_1$										

276. It will be noticed that when $D_0 - D_1 = 0$, $C = 0$. Therefore for all values of $c_0 - c_1$, the lines pass through the origin.

We may proceed to plat the lines $c_0 - c_1 = 1$, $c_0 - c_1 = 2$, $c_0 - c_1 = 3$, etc., from data shown in the above table, platting upon the lines $D_0 - D_1 = 3.24$, $D_0 - D_1 = 6.48$, etc., the points shown with circles around them in the cross-section sheet, p. 189.

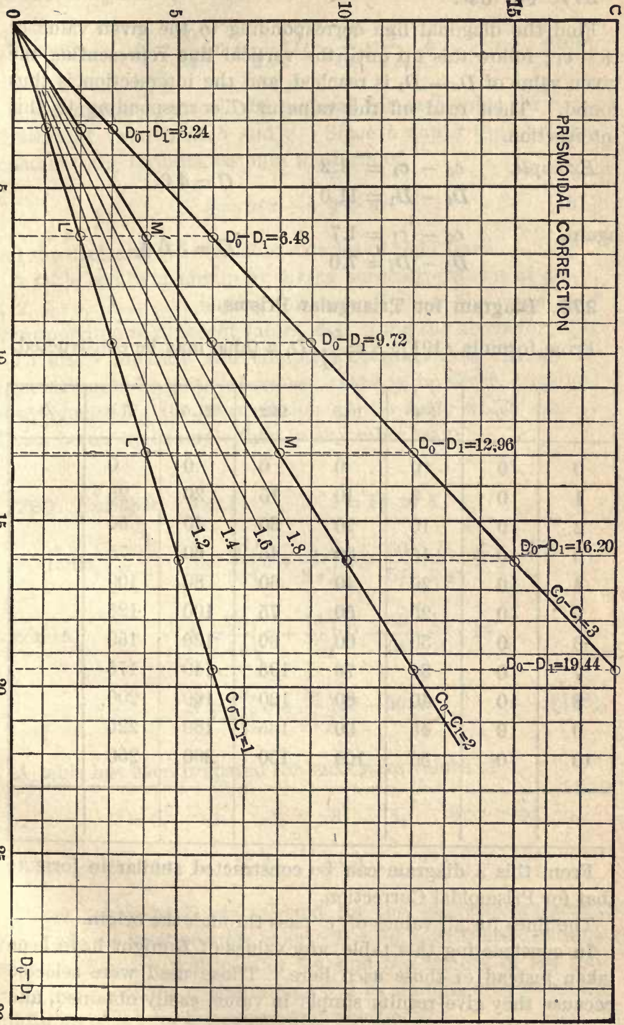
Having the lines $c_0 - c_1 = 1$, $c_0 - c_1 = 2$, 3, platted, intermediate lines are interpolated mechanically upon the principle that *vertical* lines would be proportionally divided (as ML is proportionally divided into 5 equal parts), and points are marked for the lines

$$c_0 - c_1 = 1.2, \quad 1.4, \quad 1.6, \quad 1.8$$

For the most convenient use, the values of $c_0 - c_1$ are taken to every second tenth of a foot in interpolating, as is shown on the diagram, p. 189, between 1 and 2; that is,

1.2, 1.4, 1.6, 1.8

A complete diagram is shown at the back of the book.


PRISMOIDAL CORRECTION
D0−D1=3.24
D0−D1=6.48
D0−D1=9.72
D0−D1=12.96
D0−D1=16.20
D0−D1=19.44
C0−C1=3
C0−C1=2
C0−C1=1
1.2
1.4
1.6
1.8
L
M
L′
M′
0
5
10
15
20
25
30
D0−D1

277. For Use.

Find the diagonal line corresponding to the given value of $c_0 - c_1$; follow this up until the vertical line representing the given value of $D_0 - D_1$ is reached, and the intersection is thus found. Then read off the value of C corresponding to this intersection.

Example. $c_0 - c_1 = 1.2$, $D_0 - D_1 = 11.0$ $\qquad C = 4.0$

again, $c_0 - c_1 = 1.7$, $D_0 - D_1 = 7.0$ $\qquad C = 3.6 \pm$

278. Diagram for Triangular Prisms.

From formula (191), $S = \frac{50}{54} cD$, a table may be constructed.

	0	5.4	10.8	16.2	21.6	27.0	*D*
0	0	0	0	0	0	0	
1	0	5	10	15	20	25	
2	0	10	20	30	40	50	
3	0	15	30	45	60	75	
4	0	20	40	60	80	100	
5	0	25	50	75	100	125	
6	0	30	60	90	120	150	
7	0	35	70	105	140	175	
8	0	40	80	120	160	200	
9	0	45	90	135	180	225	
10	0	50	100	150	200	250	
c							

From this a diagram can be constructed similar in form to that for Prismoidal Correction.

The lines for all values of c pass through the origin.

In constructing this table, any values of D might have been taken instead of those used here. Those used were selected because they give results simple in value, easily obtained, and readily platted.

279. Diagram for Three-Level Sections.

Formula, $$S = \frac{50}{54}\left(c + \frac{b}{2\,s}\right)D - \frac{50}{54} \cdot \frac{b}{2\,s} \cdot b \qquad (192)$$

A separate diagram will be required for each value (or combination of values) of b and s. Since b and s thus become constants, the formula assumes the form of

$$x = a(z + b)y + d \qquad (197)$$

and the diagram will consist of a series of right lines.

A table can be made up by taking successive values of $c = 0$, 1, 2, 3, 4, etc., and finding for each of these the value of S corresponding to different values of D, using the above formula.

To make separate and complete computations directly by formula would be quite laborious; there is, however, a method of systematizing the construction of the *table* which can be shown better by example than in any other way.

280. *Example.* $b = 14$ $\qquad s = 1\tfrac{1}{2}$ to 1

Formula $$S = \frac{50}{54}\left(c + \frac{b}{2\,s}\right)D - \frac{50}{54} \cdot \frac{b}{2\,s} \cdot b$$

becomes $$S = \frac{50}{54}\left(c + \frac{14}{3}\right)D - \frac{50}{54} \cdot \frac{14}{3} \cdot 14$$

$$S = \frac{50}{54}\left(c + \frac{14}{3}\right)D - 60.49 \qquad (198)$$

A table has been prepared for successive values of

$c = 0$, 1, 2, 3, 4, 5, etc.

and for $D = 14$, 16.2, 21.6, 27.0, etc.

These values of D are selected for the following reasons: $D = 14$ is the least possible value; $D = 16.2$, 21.6 are desirable because they are multiples of 5.4, and the factors in the formula show that the computations will be simplified by selecting multiples of 5.4 for the successive values of D

	14	16.2	21.6	27.0	32.4	37.8	43.2	*D*
	12.963	15.	20.	25.	30.	35.	40.	Const. diff.
0	0	9.51	32.84	56.18	79.51	102.84	126.18	
1	12.963	24.51	52.84	81.18	109.51	137.84	166.18	
2	25.926	39.51	72.84	106.18	139.51	172.84	206.18	
3	38.889	54.51	92.84	131.18	169.51	207.84	246.18	
4	51.852	69.51	112.84	156.18	199.51	242.84	286.18	
5	64.815	84.51	132.84	181.18	229.51	277.84	326.18	
6	77.778	99.51	152.84	206.18	259.51	312.84	366.18	
7	90.741	114.51	172.84	231.18	289.51	347.84	406.18	
8	103.704	129.51	192.84	256.18	319.51	382.84	446.18	
9	116.667	144.51	212.84	281.18	349.51	417.84	486.18	
10	129.630	159.51	232.84	306.18	379.51	452.84	526.18	
c								

When $\quad c = 0 \qquad S = \frac{50}{54} \cdot \frac{14}{3} \cdot D - 60.49$

When $\quad D = 14 \qquad S = \frac{50}{54} \cdot \frac{14}{3} \cdot 14 - 60.49$

$$= 60.49 \quad - 60.49 = 0$$

When $\quad D = 16.2$

we may again calculate directly

$$S = \frac{50}{54} \cdot \frac{14}{3} \cdot 16.2 - 60.49$$

but a better method is to find how much greater S will be for $D = 16.2$ than for $D = 14.0$.

We have $\quad S = \frac{50}{54} \cdot \frac{14}{3} \cdot D - 60.49$

Then for any new value D'

$$S' = \frac{50}{54} \cdot \frac{14}{3} \cdot D' - 60.49$$

$$S' - S = \frac{50}{54} \cdot \frac{14}{3}(D' - D) \tag{199}$$

for $\quad D' = 16.2 \qquad D = 14.0 \qquad D' - D = 2.2$

$$S' - S = \frac{50}{54} \cdot \frac{14}{3} \times 2.2 = 9.51$$

$$S = 0$$

$S' = 9.51$, which is entered in table.

Similarly, $S'' - S' = \frac{50}{54} \cdot \frac{14}{3}(D'' - D')$

$$D'' = 21.6 \qquad D' = 16.2 \qquad D'' - D' = 5.4$$

$$S'' - S' = \frac{50}{54} \times \frac{14}{3} \times 5.4$$

$$= 23.333$$

$$S' = 9.51 \qquad S^{iv} = 79.509$$

$$S'' = 32.843 \qquad 23.333$$

Similarly, $S''' - S'' = 23.333 \qquad S^{v} = 102.842$

$$S''' = 56.176 \qquad 23.333$$

$$S^{iv} - S''' = 23.333 \qquad S^{vi} = 126.175$$

$$S^{iv} = 79.509$$

Constant increment for $D' - D = 5.4$ is 23.333.

281. Each result is entered in the table in its proper place. The final result for $c = 0$ and $D = 43.2$ should be calculated independently as a check.

When $c = 0$ $\qquad S = \frac{50}{54} \cdot \frac{14}{3} \cdot D - 60.49$

When $D = 43.2$ $\qquad S = \frac{50}{54} \cdot \frac{14}{3} \times 43.2 - 60.49$

$$= 50 \times \frac{14}{3} \times 0.8 - 60.49$$

$$= \frac{560}{3} - 60.49$$

$$= 186.67 - 60.49$$

$$S = 126.18$$

This checks exactly, and all intermediate values are checked by this process, which is also more rapid than an independent calculation for each value of D.

282. We now have values of S for the various values of $D = 14.0$, 16.2, 21.6, etc., when $c = 0$.

Next, find how much these will be increased when $c = 1$.

Formula $\qquad S = \frac{50}{54}(c + \frac{14}{3})D - 60.49$

for any new value c' $\qquad S' = \frac{50}{54}(c' + \frac{14}{3})D - 60.49$

$$S' - S = \frac{50}{54}(c' - c)D \qquad (200)$$

When $c' = 1$ and $c = 0$, $\quad c' - c = 1$

$$S' - S = \tfrac{50}{54} D$$

Similarly, $\quad S'' - S' = \tfrac{50}{54}(c'' - c')D$

When $c'' = 2$ and $c' = 1$, $\quad c'' - c' = 1$

$$S'' - S' = \tfrac{50}{54} D$$

That is, for *any increase* of 1 ft. in the value of c,

$$S' - S = \tfrac{50}{54} D \tag{201}$$

When $D = 14$

$$S' - S = \tfrac{50}{54} \times 14 = 12.963$$

This we enter as the constant difference for column $D = 14$.

We have already found $\quad S_0 = 0$

$$\begin{aligned} S_0 &= 0 \\ &\underline{\;12.963} \\ S_1 &= 12.963 \\ &\underline{\;12.963} \\ S_2 &= 25.926 \end{aligned}$$

This gives column 14. $\quad S_3 = 38.889$ etc.

When $D = 16.2$

(201) $\quad S' - S = \tfrac{50}{54} D = \tfrac{50}{54} \times 16.2 = 50 \times 0.3$
$= 15$

Enter 15 as the constant difference in column 16.2.

We already have

$$\begin{aligned} S_0 &= 9.51 \\ &\underline{\;15.} \\ S_1 &= 24.51 \\ &\underline{\;15.} \\ S_2 &= 39.51 \end{aligned}$$

This allows us to complete column 16.2. $\quad S_3 = 54.51$ etc.

Similarly for $D = 21.6 \qquad S' - S = 20$

Enter 20 as constant difference in column 21.6, and complete column as shown in table.

Similarly, fill out all the columns shown in the table.

283. The final result for $c = 10$, $D = 43.2$ should be calculated independently, and directly from the formula, as a check.

$$S = \tfrac{50}{54}(c + \tfrac{14}{3})D \qquad - 60.49$$

$$c = 10 \qquad D = 43.2$$

$$S = \tfrac{50}{54} \times 14.667 \times 43.2 - 60.49$$

$$= 50 \times 14.667 \times 0.8 - 60.49$$

$$= 40 \times 14.667 \qquad - 60.49$$

$$= 586.68 \qquad - 60.49$$

$$S = 526.19$$

The table gives 526.18. This checks sufficiently close to indicate that no error has been made. It would yield an exact check if we took $c + \frac{14}{3} = 14.6667$.

284. Note that for	$c = 10$	$D = 43.2$	value $= 526.18$
	$c = 10$	$D = 37.8$	" 452.84
			Diff. $= 73.34$
Between	$c = 10$	$D = 37.8$	Diff. $= 73.33$
and	$c = 10$	$D = 32.4$	

In line $c = 10$ a constant difference is found between successive values of D differing by 5.4. This may be demonstrated to be $= 73.33$.

All values in the table except column 14 are satisfactorily checked by applying this difference of 73.33 in line 10 together with the independent check of $c = 10$, $D = 43.2$.

The value of $c = 10$, $D = 14$ can also be checked and shown to be correct.

285. Having the table, page 192, completed, the construction of the diagram is simple.

The "*Diagram for Three-Level Sections, Base* 14, *Slope* $1\frac{1}{2}$ *to* 1," was calculated and constructed according to this table. The Diagram given shows a general arrangement of lines and figures convenient for use. For rapid and convenient use, the diagram should be constructed upon cross-section paper, Plate G; and in this case the diagram will be upon a scale twice that of the diagram accompanying these notes.

A "curve of level section" has been platted on this diagram in the following manner. For level sections, when

$c = 0$	$D = 14.0$	$c = 2$	$D = 20.0$
$c = 1$	$D = 17.0$	$c = 6$	$D = 32.0$
$c = 1.4$	$D = 18.2$ etc.		

The line passing through these points gives the "curve of level section."

Aside from the direct use of this curve of level section (for preliminary estimates or otherwise), it is very useful in tending to prevent any gross errors in the use of the table, since, in general, the points (intersections) used in the diagram will lie not far from the curve of level section.

286. Use of Diagram.

Find the diagonal line corresponding to the given value of c; follow this up until the vertical line representing the given value of D is reached, and this intersection found. Then read off the value of S corresponding to this intersection.

Example. Notes.

Sta. 1	$\frac{16.0}{-6.0}$	-3.7	$\frac{12.4}{-3.6}$	$S_1 = 160.$
Sta. 0	$\frac{10.6}{-2.4}$	-2.5	$\frac{10.3}{-2.2}$	$S_0 = 78.$
				$V = 238.$

For Sta. 1 $c = 3.7$ $D = 28.4$

$c = 3.7$ is the middle of the space between 3.6 and 3.8.
Follow this up until the vertical line 28.4 is reached.
The intersection lies upon the line $S_1 = 160$.
Enter this above opposite Sta. 1.

For Sta. 0 $c = 2.5$ $D = 20.9$

$c = 2.5$ is the middle of space between 2.4 and 2.6.
Follow this up until the middle of space between 20.8 and 21.0 is reached.
The intersection lies just above the line

$$S_0 = 78$$

Enter this opposite Sta. 0.

$$V_{100} = S_1 + S_0$$
$$= 160 + 78 = 238 \text{ cu. yds.}$$

The prismoidal correction may be applied if desired.

287. Diagrams may be constructed in this way that will give results to a greater degree of precision than is warranted by the precision reached in taking the measurements on the ground.

In point of rapidity *diagrams are much more rapid than tables* for the computation of *Three-Level Sections.*

For "*Triangular Prisms*" and for *Prismoidal Correction*, the *diagrams* are *somewhat more rapid.*

For *Level Sections*, the tables for Three-Level Sections are *at least equally rapid.*

288. The use of approximate methods for applying the prismoidal correction to irregular sections will now be rendered very practicable by the use of these "Diagrams for Three-Level Sections."

Method 1. No use of diagrams is necessary.

Method 2. Having found for any irregular sections (by triangular prisms or any other method) the solidity S for 50 ft. in length, find upon the diagram the line corresponding to this value of S; follow this line to the curve of level section, and read off the value of c (for a level section) which corresponds, and also the value of D for the same section.

Method 3. Having found in any way the value of S; if c is given, find the value of D to correspond; if D is given, find the value of c to correspond.

Method 4. The use of diagrams is not needed.

The diagrams shown at the back of the book are given partly to show a good scheme or arrangement, and partly to allow practice in their use. For regular work the scale is too small to be desirable, and trying to the eyes. They are not sufficiently extensive. In offices where there is much earthwork computation to be done, diagrams should be constructed on double the scale and extending to higher numbers. Several sheets may be required for each kind of diagram. It may seem that sufficiently precise values cannot be read from these diagrams, but the diagrams are much more precise than the field-work, where a center cut is not sure to one tenth of a foot.

CHAPTER XVI.

HAUL.

289. When material from excavation is hauled to be placed in embankment, it is customary to pay to the contractor a certain sum for every cubic yard hauled. Oftentimes it is provided that no payment shall be made for material hauled less than a specified distance. In the east a common limit of "free haul" is 1000 ft. Often in the west 500 ft. is the limit of "free haul." Sometimes 100 ft. is the limit.

A common custom is to make the unit for payment of haul, one yard hauled 100 ft.; the price paid will often be from 1 to 2 cents per cubic yard hauled 100 ft.

The price paid for "haul" is small, and therefore the standard of precision in calculation need not be quite as fine as in the calculation of the quantities of earthwork. The total "haul" will be the product of

(1) the total amount of excavation hauled, and

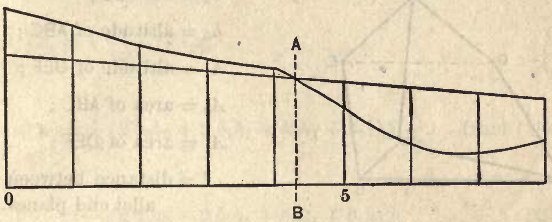
(2) the average length of haul.

290. The average length of haul is the distance between the center of gravity of the material as found in excavation, and the center of gravity as deposited. It would not, in general, be simple to find the center of gravity of the entire mass of excavation hauled, and the most convenient way is to take each section of earthwork by itself. The "haul" for each section is the product of the

(1) number of cubic yards in that section, and

(2) distance between the center of gravity in excavation, and the center of gravity as deposited.

291. When excavation is placed in embankment, there may be some difficulty in determining just where any given section of excavation will be placed, and where its center of gravity will be in embankment.



In hauling excavation in embankment, there is some plane, as indicated by AB, *to* which all excavation must be hauled on its way to be placed in embankment, and (another way of putting it) *from* which all material placed in embankment must be hauled on its way from excavation. We may figure the total haul as the sum of

(1) total "haul" of excavation *to* AB, and

(2) total "haul" of embankment *from* AB.

The total "haul" of excavation *to* AB and the total "haul" of embankment *from* AB will most conveniently be calculated as the sum of the hauls of the several sections of earthwork. For each section the haul is the product of

(1) the volume V of that section, and

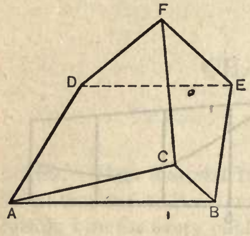
(2) distance from center of gravity of that section to the plane AB.

292. When the two end areas are equal, the center of gravity will be midway between the two end planes. When the two end areas are not equal in value, the center of gravity of the section will be at a certain distance from the mid-section, as shown by the formula

$$x_g = \frac{l^2}{12} \cdot \frac{A_1 - A_0}{V}$$

in which x_g = distance of center of gravity from mid-section.

293. Referring to the figure below, and following the same general method of demonstration used previously in § 237,



let $b_0 =$ base AB ;

$b_1 =$ base DE ;

$h_0 =$ altitude of ABC ;

$h_1 =$ altitude of DEF ;

$A_0 =$ area of ABC ;

$A_1 =$ area of DEF ;

$l =$ distance between parallel end planes.

Also use notation b_x, h_x, A_x, for a section distant x from ABC.

$x_c =$ distance of center of gravity from ABC, for entire section of earthwork.

$x_g =$ distance of center of gravity from midsection.

Then for any elementary section of thickness dx, and distance x from ABC, its moment will be

$$\frac{1}{2}\left[b_0 - (b_0 - b_1)\frac{x}{l}\right]\left[h_0 + (h_1 - h_0)\frac{x}{l}\right] x\, dx$$

$$V \cdot x_c = \int_0^l \frac{1}{2}\left[b_0 + (b_1 - b_0)\frac{x}{l}\right]\left[h_0 + (h_1 - h_0)\frac{x}{l}\right] x\, dx$$

$$= \frac{b_0 h_0 l^2}{4} + \frac{b_0(h_1 - h_0)\, l^3}{6\, l} + \frac{h_0(b_1 - b_0)\, l^3}{6\, l} + \frac{(b_1 - b_0)(h_1 - h_0)\, l^4}{8\, l^2}$$

$$= \frac{l^2}{24}\left[\begin{array}{l} + 6\, b_0 h_0 + 4\, b_0 h_1 + 4\, b_1 h_0 + 3\, b_1 h_1 \\ - 4\, b_0 h_0 - 3\, b_0 h_1 - 3\, b_1 h_0 \\ - 4\, b_0 h_0 \\ + 3\, b_0 h_0 \end{array}\right]$$

$$V \cdot x_c = \frac{l^2}{24} \times (b_0 h_0 + b_0 h_1 + b_1 h_0 + 3\, b_1 h_1)$$

$$x_c = \frac{l^2}{24} \times \frac{b_0 h_0 + b_0 h_1 + b_1 h_0 + 3\, b_1 h_1}{V} \tag{202}$$

What is wanted is x_g rather than x_c

$$x_g = \frac{l}{2} - x_c$$

$$V \cdot x_g = V \cdot \frac{l}{2} - V \cdot x_c$$

$$V = \frac{l}{12}(2\, b_0 h_0 + 2\, b_1 h_1 + b_0 h_1 + b_1 h_0) \qquad \text{from (164)}$$

$$V \cdot \frac{l}{2} = \frac{l^2}{24}(2\, b_0 h_0 + 2\, b_1 h_1 + b_0 h_1 + b_1 h_0) \qquad (203)$$

$$V \cdot x_c = \frac{l^2}{24}(b_0 h_0 + 3\, b_1 h_1 + b_0 h_1 + b_1 h_0)$$

$$V \cdot x_g = \frac{l^2}{24}(b_0 h_0 - b_1 h_1)$$

$$= \frac{l^2}{12}(A_0 - A_1)$$

$$x_g = \frac{l^2}{12} \frac{A_0 - A_1}{V} \qquad (V \text{ in cu. ft.}) \qquad (204)$$

$$x_g = \frac{l^2}{12 \times 27} \frac{A_0 - A_1}{V} \qquad (V \text{ in cu. yds.}) \qquad (205)$$

This formula applies directly to solids with triangular ends and with two of the surfaces either plane or warped in the manner suggested in § 236. Regular Three Level Sections may be divided into parts of triangular section, so that the above formula will apply in that case. In a similar way it will apply to Irregular Sections as in § 229, or to sections even more irregular as on pages 176, 177.

295. The formula

$$x_g = \frac{l^2}{12 \times 27} \cdot \frac{A_1 - A_0}{V}$$

is not in form convenient for use, because we have not found the values of A_1 and A_0, but instead have calculated directly from the tables or diagrams the values of S_1 and S_0 for 50 ft. in length, where

$$S_1 = \frac{50}{27} A_1, \text{ or } A_1 = \frac{27\ S_1}{50}$$

and

$$A_0 = \frac{27}{50} S_0$$

Substituting, $$x_{g_{100}} = \frac{100 \times 100}{12 \times 27} \cdot \frac{S_1 - S_0}{V} \cdot \frac{27}{50}$$

$$x_{g_{100}} = \frac{100}{6} \cdot \frac{S_1 - S_0}{V} \qquad (206)$$

where V is the correct volume in cu. yds.

This formula is in shape convenient for use, and results correct to the nearest foot can be calculated with rapidity.

296. For a section of length l less than 100 ft.

$$x_{g_l} = \frac{l^2}{12 \times 27} \cdot \frac{A_1 - A_0}{V_l}$$

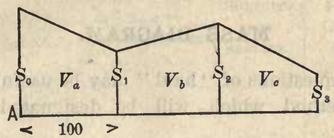
$$= \frac{l^2}{12 \times 27} \cdot \frac{A_1 - A_0}{V_{100} \times \frac{l}{100}}$$

$$= \frac{100\ l}{12 \times 27} \cdot \frac{A_1 - A_0}{V_{100}}$$

$$x_{g_{100}} = \frac{100 \times 100}{12 \times 27} \cdot \frac{A_1 - A_0}{V_{100}}$$

$$x_{g_l} = x_{g_{100}} \cdot \frac{l}{100} \qquad (207)$$

297. For a *series of sections*, each 100 ft., a correction may be applied to obtain the correction in haul for the entire mass.



Let X_c = dist. from A to *c.g.* for entire mass (approx.), found by using for each section *c.g.* at $\frac{l}{2}$

H_a = approx. total haul from A $= V \times X_c$

X = true dist. to *c.g.* of entire mass from A

H = correct total haul $= V \times X$

$S_0 = \frac{50}{27} A_0$, $S_1 = \frac{50}{27} A_1$, S_2 = etc., as taken from tables or diagrams.

When all sections are of uniform length, 100′ as in figure above, the approximate total haul for the figure above

$$H_a = X_c V = \frac{100}{2}(V_a \qquad + 3\,V_b \qquad + 5\,V_c)$$

Correct total haul

$$H = XV = V_a\left(\frac{100}{2} - x_{ga}\right) + V_b\left(3\frac{100}{2} + x_{gb}\right) + V_c\left(5\frac{100}{2} - x_{gc}\right)$$

$$H_a - H = \qquad V_a x_{ga} \quad - \quad V_b x_{gb} \qquad + V_c x_{gc}$$

$$= \frac{100}{6}\left[V_a \frac{S_0 - S_1}{V_a} - \quad V_b \frac{S_2 - S_1}{V_b} \quad + V_c \frac{S_2 - S_3}{V_c}\right]$$

$$= \frac{100}{6}(S_0 - S_3)$$

Or for a more general and more convenient form, reduced to cu. yds. hauled 100 ft.

$$H_a - H = \tfrac{1}{6}(S_0 - S_n) = \text{correction in total haul.} \qquad (208)$$

While it is correct to apply formula (208) with the proper algebraic signs, it is simpler to compute the numerical value of $H_a - H$; then the true center of gravity of the entire mass will lie nearer to the larger of the extreme end sections than does the approximate center of gravity.

CHAPTER XVII.

MASS DIAGRAM.

298. Many questions of "haul" may be usefully treated by a graphical method which will be designated the "Mass Diagram."

The construction of the "Mass Diagram" will be more clearly understood from an example than from a general description.

Consider the earthwork shown by the profile on p. 206, consisting of alternate "cut" and "fill." To show the work of constructing the "diagram" in full, the quantities are calculated throughout, but for convenience, "level sections" are used and prismoidal correction disregarded. In actual practice, the solidities will have been calculated for the actual notes taken. Allowance should be made for the fact that earth placed in fill shrinks. The allowance to be made in column 5 of table will depend on how the work is to be handled. In column 5 opposite, it is assumed that, without changing the notes, additional material is placed in the fill to provide for shrinkage or settlement, which accords with common practice; and 5 per cent shrinkage is used here.

299. In the table, p. 205, columns 1 and 2 explain themselves.

3d column gives values of S from tables.

4th column gives values of S_{100} or S_l for each section, and with sign $+$ for cut or $-$ for fill.

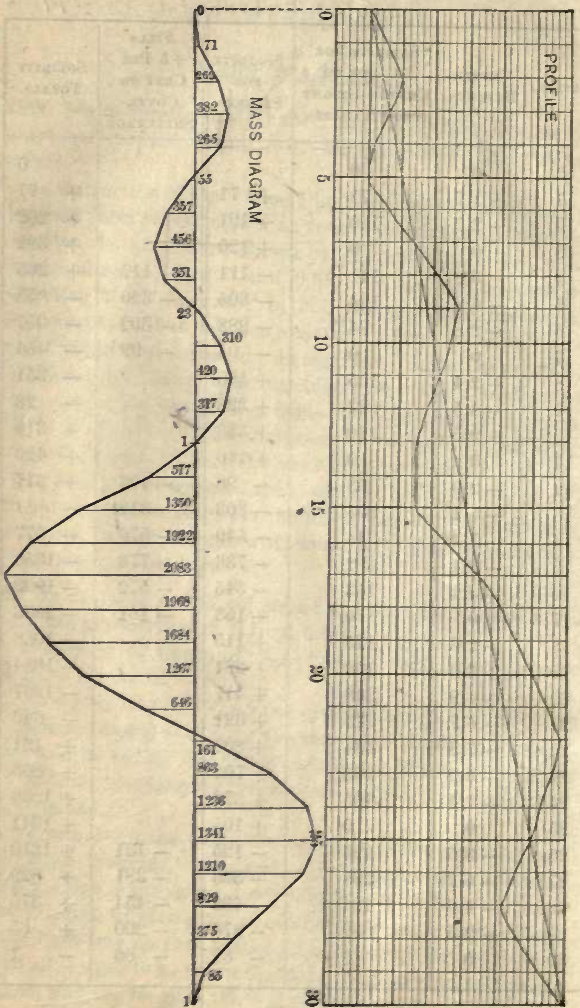
5th column shows fills after 5 per cent shrinkage.

6th column gives the total, or the sum of solidities up to each station; and in getting this total, each $+$ solidity is added and each $-$ solidity is subtracted, as appears in the table from the results obtained.

Having completed the table, the next step is the construction of the "Mass Diagram," page 206. In the figure shown there, each station line is projected down, and the value from column 6, corresponding to each station, is platted to scale as an offset from the base line at that station, all $+$ quantities above the line, and all $-$ quantities below the line. The points thus found are joined, and the result is the "Mass Diagram."

p 226 Excavation = Base of 20' in ta
p 222 Embankment = " " 14' " "

STATION.	CENTER HEIGHTS.	SOLIDITY FOR 50' DUE TO CENTER HEIGHT (FROM TABLES).	SOLIDITY FOR SECTION.	FILLS + 5 PER CENT TO COVER SHRINKAGE	SOLIDITY TOTALS.
0	0	0			0
1	+ 1.7	71	+ 71		+ 71
2	+ 2.7	120	+ 191		+ 262
3	0	0	+ 120		+ 382
4	− 3.2	111	− 111	− 117	+ 265
5	− 4.9	194	− 305	− 320	− 55
6	− 2.8	94	− 288	− 302	− 357
7	0	0	− 94	− 99	− 456
8	+ 2.4	105	+ 105		− 351
9	+ 4.5	223	+ 328		− 23
10	+ 2.5	110	+ 333		+ 310
11	0	0	+ 110		+ 420
12	− 2.9	98	− 98	− 103	+ 317
13	− 5.1	205	− 303	− 318	− 1
14	− 7.4	344	− 549	− 576	− 577
15	− 8.1	392	− 736	− 773	− 1350
16	− 4.1	153	− 545	− 572	− 1922
17	0	0	− 153	− 161	− 2083
18	+ 2.6	115	+ 115		− 1968
19	+ 3.6	169	+ 284		− 1684
20	+ 4.9	248	+ 417		− 1267
21	+ 6.7	373	+ 621		− 646
22	+ 7.5	434	+ 807		+ 161
23	+ 5.2	268	+ 702		+ 863
24	+ 2.4	105	+ 373		+ 1236
25	0	0	+ 105		+ 1341
26	− 3.5	125	− 125	− 131	+ 1210
27	− 5.7	238	− 363	− 381	+ 829
28	− 4.9	194	− 432	− 454	+ 375
29	− 2.5	82	− 276	− 290	+ 85
30	0	0	− 82	− 86	− 1


MASS DIAGRAM
PROFILE
0
5
10
15
20
30
71
262
382
265
55
357
456
351
23
310
420
317
1
577
1350
1922
2083
1968
1684
1267
646
161
563
1236
1341
1210
829
375
85
1

300. It will follow, from the methods of calculation and construction used, that the "Mass Diagram" will have the following properties, which can be understood by reference to the profile and diagram, page 206.

1. Grade points of the profile correspond to maximum and minimum points of the diagram.

2. In the diagram, ascending lines mark excavation, and descending lines embankment.

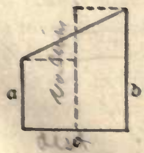
3. The difference in length between any two vertical ordinates of the diagram is the solidity between the points (stations) at which the ordinates are erected.

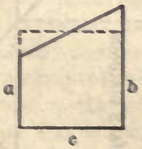
4. Between any two points where the diagram is intersected by any horizontal line, excavation equals embankment.

5. The area cut off by any horizontal line is the measure of the "haul" between the two points cut by that line.

301. It may be necessary to explain the latter point at somewhat greater length.

Any quantity (such as dimension, weight, or volume) is often represented graphically by a line; in a similar way, the product of two quantities (such as volume into distance, or as foot pounds) may be represented or measured by an area. In the case of a figure other than a rectangle, the value, or product measured by this area, may be found by cutting up the area by lines, and these lines may be vertical lines representing volumes or horizontal lines representing distance. The result will be the same in either case. An example will illustrate.



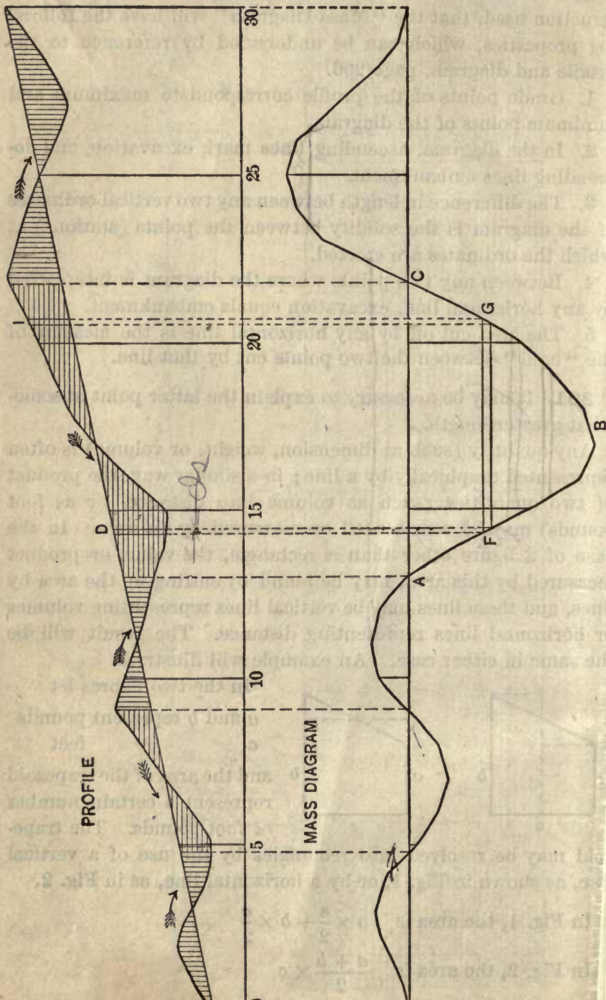


In the two figures let a and b represent pounds, c " feet and the area of the trapezoid represent a certain number of foot pounds. The trapezoid may be resolved into rectangles by the use of a vertical line, as shown in Fig. 1, or by a horizontal line, as in Fig. 2.

In Fig. 1, the area is $a \times \frac{c}{2} + b \times \frac{c}{2}$

In Fig. 2, the area is $\frac{a+b}{2} \times c$

the result of course being the same in both cases.


PROFILE
MASS DIAGRAM
0
5
10
15
20
25
30
A
B
C
D
F
G
I

S = Solidity

302. In an entirely similar way, the area ABC (p. 208) represents the "haul" of earthwork (in cu. yds. moved 100 ft.) between A and C, and this area may be calculated by dividing it by a series of vertical lines representing solidities, as shown above G and F. That this area represents the haul between A and C may be shown as follows: —

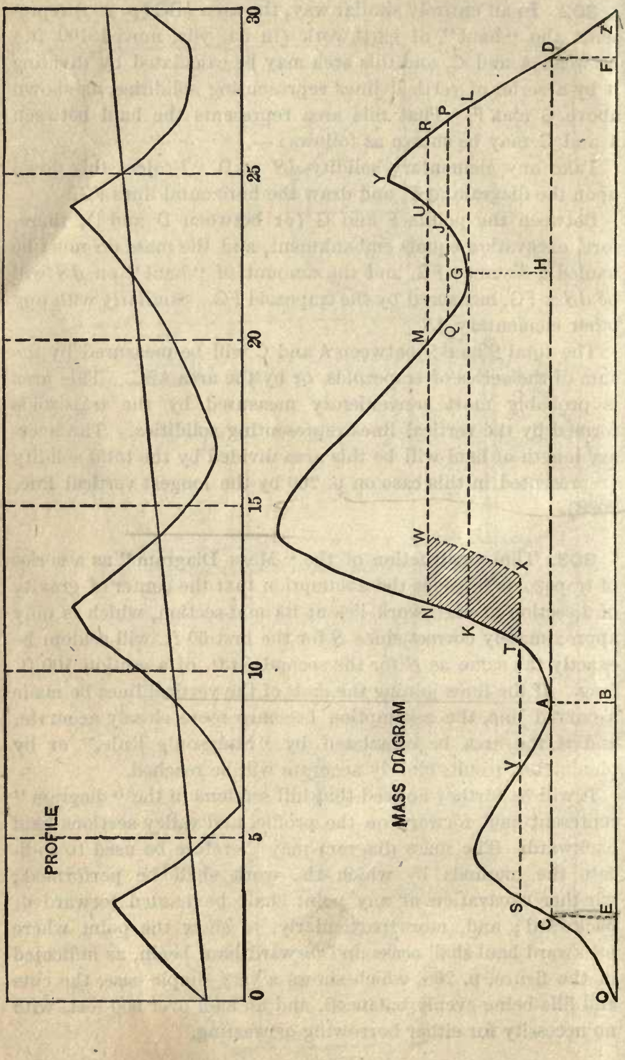
Take any elementary solidity dS at D. Project this down upon the diagram at F, and draw the horizontal lines FG.

Between the points F and G (or between D and I), therefore, excavation equals embankment, and the mass dS must be hauled a distance FG, and the amount of "haul" on dS will be $dS \times$ FG, measured by the trapezoid FG. Similarly with any other elementary dS.

The total "haul" between A and C will be measured by the sum of the series of trapezoids, or by the area ABC. This area is probably most conveniently measured by the trapezoids formed by the vertical lines representing solidities. The average length of haul will be this area divided by the total solidity (represented in this case on p. 206 by the longest vertical line, 2083).

303. The construction of the "Mass Diagram" as a series of trapezoids involves the assumption that the center of gravity of a section of earthwork lies at its mid-section, which is only approximately correct since S for the first 50 ft. will seldom be exactly the same as S for the second 50 ft. of a section 100 ft. long. If the lines joining the ends of the vertical lines be made a curved line, the assumption becomes more closely accurate, and if the area be calculated by "Simpson's Rule," or by planimeter, results closely accurate will be reached.

It will be further noticed that hill sections in the "diagram" represent haul forward on the profile, and valley sections haul backward. The mass diagram may therefore be used to indicate the methods by which the work shall be performed; whether excavation at any point shall be hauled forward or backward; and, more particularly, to show the point where backward haul shall cease and forward haul begin, as indicated in the figure, p. 208, which shows a very simple case, the cuts and fills being evenly balanced, and no haul over 900 feet, with no necessity for either borrowing or wasting.


PROFILE
MASS DIAGRAM
0
5
10
15
20
25
30
O C E S V A B T K N X W M Q G H J U Y R P L D F Z

304. In the figure, page 210, the excavation from Sta. 0 to 14 is very much in excess of embankment, and *vice versa* from Sta. 14 to 30. The mass diagram indicates a haul of nearly 3000 ft. for a large mass of earthwork, measured by the ordinate AB. It will not be economical to haul the material 3000 ft.; it is better to "waste" some of the material near Sta. 0, and to "borrow" some near Sta. 30, if this be possible, as is commonly the case.

If we draw the line CD, the cut and fill between C and D will still be equal, and the volume of cut measured by CE can be wasted, and the equal volume of fill measured by DF can be borrowed to advantage. It can be seen that there is still a haul of nearly 2000 ft. (from A to D) on the large mass of earthwork measured by GH. It is probable that it will not pay to haul the mass GH, or any part of it, as far as AD.

305. We must find the limit beyond which it is unprofitable to *haul* material rather than *borrow* and *waste*.

Let c = cost of 1 cu. yd. excavation or embankment.

h = cost of haul on 1 cu. yd. hauled 100 ft.

n = length of haul in "stations" of 100 ft. each.

Then, when 1 cu. yd. of excavation is wasted, and 1 cu. yd. of embankment is borrowed,

$$\text{cost} = 2\,c$$

When 1 cu. yd. of excavation is hauled into embankment,

$$\text{cost} = c + nh$$

The limit of profitable haul is reached when

$$2\,c = c + nh$$

or when

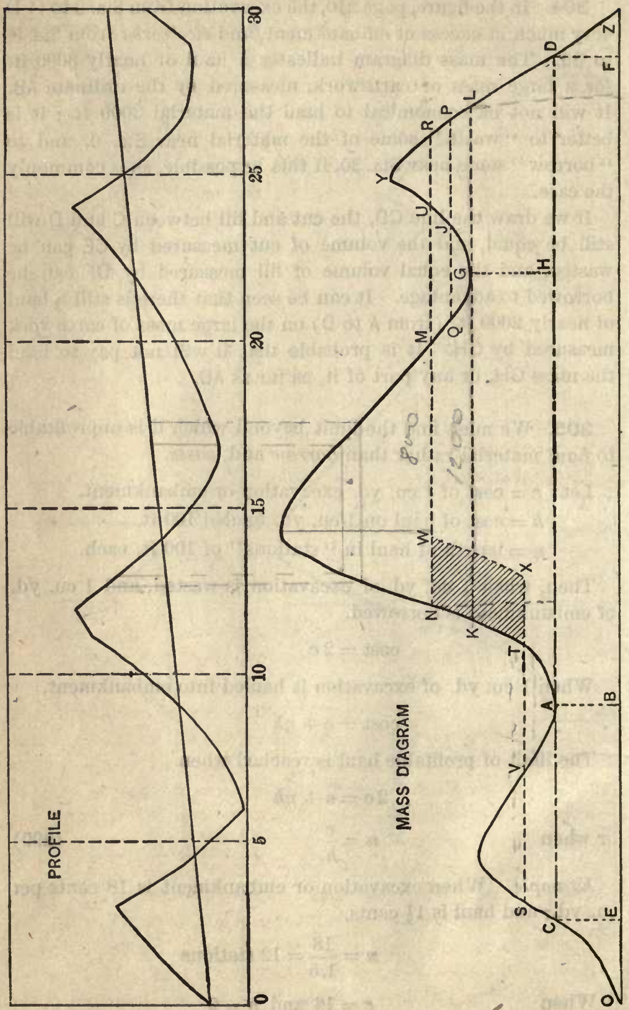
$$n = \frac{c}{h} \tag{209}$$

Example. When excavation or embankment is 18 cents per cu. yd., and haul is $1\frac{1}{2}$ cents,

$$n = \frac{18}{1.5} = 12 \text{ stations}$$

When $c = 16$ and $h = 2$

then $n = 8$ stations


PROFILE
MASS DIAGRAM
0
5
10
15
20
25
30
O
C
E
S
V
A
B
T
K
N
X
W
M
Q
G
H
J
U
Y
R
P
L
D
F
Z
800
1200

306. In the former case (1200 ft. haul) we should draw in mass diagram (p. 212) the line KGL. Here KG is less than 1200 ft. The line should not be lower than G, for in that case the haul would be nearly as great as KL, or more than 1200 ft.

In the latter case (800 ft. haul) the line would be carried up to a point where NM = 800 ft. The masses between N and A, also C and O, can better be wasted than hauled, and the masses between M and G, also L and Z, can better be borrowed than hauled (always provided that there are suitable places at hand for borrowing and wasting).

Next, produce NM to R. The number of yards borrowed will be the same whether taken at RZ or at MG + LZ. That arrangement of work which gives the smallest "haul" (product of cu. yds. × distance hauled) is the best arrangement. The "haul" in one case is measured by GLRYG, and in the other by MGU + UYR. If MGU is less than GLRU, then it is cheaper to borrow (*a*) RZ rather than (*b*) MG + LZ. The most economical position for the line is when QJ = JP. For any change from this position will show an increase of net area representing "haul."

In a similar way NT and SO can be more economically wasted than NA and CO. Here make SV = VT.

307. The case is often not as simple as that here given. Very often the material borrowed or wasted has to be hauled beyond the limit of "free haul." The limit beyond which it is unprofitable to haul will vary according to the length of haul on the borrowed or wasted material; the limit will, in general, be increased by the length of haul on the borrowed or wasted material. The haul on wasted or borrowed material, as NT, may be shown graphically by NTXW, where NW = TX shows the length of haul, and NTXW the "haul" (mass × distance).

The mass diagram can be used also for finding the limit of "free haul" on the profile, and various applications will suggest themselves to those who become familiar with its use and the principles of its construction. Certainly one of its most important uses is in allowing "haul" and "borrow and waste" to be studied by a diagram giving a comprehensive view of the whole situation. There are few if any other available methods of accomplishing this result.

308. When material is first taken out in excavation, it generally occupies more space than was originally the case. When placed in embankment, it commonly shrinks somewhat and eventually occupies less space than originally. Wherever, from any cause, the material put into embankment will occupy more space or less space than it did in excavation, the quantities in embankment should be corrected before figuring haul or constructing a Mass Diagram, and a column should be shown for this as is done in Table p. 205.

309. Many engineers write their contracts and specifications without a clause allowing payment for "haul" or "overhaul." Nevertheless it appears that it is the more common practice to insert a clause providing for payment for overhaul. A canvass on this subject by the American Railway Engineering and Maintenance of Way Association in 1905 showed this practice to prevail in the proportion of 73 to 37. The free haul limit of 500 ft. seemed to meet with greater favor than any other.

Where an "overhaul" clause is inserted in a contract, the basis of payment has varied on different railroads. In one method, not recommended, the total haul is to be computed; from this shall be deducted for free haul the total "yardage" multiplied by the length of the free haul limit. Under this system, with a 500 ft. free haul limit, there might be 10,000 cu. yds. of earth hauled (all of it) more than 500 ft., or an average of 600 ft.; yet if there were another 10,000 cu. yds. hauled an average of 300 ft., there would be no payment whatever for overhaul; the average haul would be less than 500 ft. Unless the specifications clearly show that this method is to be used, it is unfair as well as unsatisfactory to the contractor.

What seems a logical and satisfactory provision is that recommended by the American Railway Engineering and Maintenance of Way Association by a letter ballot vote of 134 to 23 (announced in March, 1907). This is as follows: —

"No payment will be made for hauling material when the length of haul does not exceed the limit of free haul, which shall be _____ feet.

"The limits of free haul shall be determined by fixing on the profile two points, one on each side of the neutral grade point, one in excavation and the other in embankment, such that the distance between them equals the specified free haul limit, and

the included quantities of excavation and embankment balance. All haul on material beyond this free haul limit will be estimated and paid for on the basis of the following method of computation, viz.: —

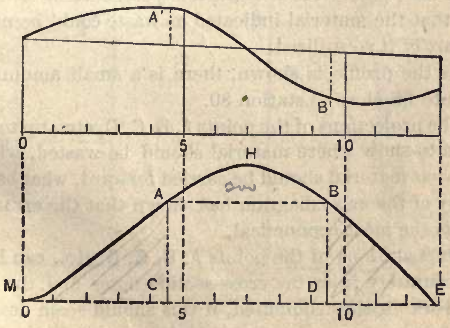
"All material within this limit of free haul will be eliminated from further consideration.

"The distance between the center of gravity of the remaining mass of excavation and center of gravity of the resulting embankment, less the limit of free haul as above described, shall be the length of overhaul, and the compensation to be rendered therefor will be determined by multiplying the yardage of the remaining mass as above described, by the length of the overhaul. Payment for the same will be by units of one cubic yard hauled one hundred feet.

"When material is obtained from borrow-pits along the embankment, and runways are constructed, the haul shall be determined by the distance the team necessarily travels. The overhaul on material thus hauled shall be determined by multiplying the yardage so hauled by one half the round distance made by the team less the free haul distance. The runways will be established by the engineer."

310. This statement as to the method of figuring overhaul is explained very simply by the Mass Diagram below. The length of AB is that of the free haul limit (500 ft. in figure). The free haul is shown in the area ACDBHA. The amount of overhaul to be paid for is shown in 2 parts, ACM, BDE.

PROFILE



MASS DIAGRAM

311. The diagram on the page opposite shows a sketch of a profile and the corresponding mass diagram ; illustrating further the method of studying questions of haul, borrow, and waste. For this purpose it is assumed that the limit of economical haul is 1000 ft., and the lines on the mass diagram are adjusted accordingly.

(*a*) Line AB = 1000 ft. and can go no lower because the limit of 1000 ft. would be exceeded; nor higher because the waste near A and the borrow near B would be increased.

(*b*) Line CDE is placed so that CD = DE ; the sum of the two borrows (between B and C, and between E and F) is the same for any practical position of CDE ; the sum of the two areas CRD and DSE is a minimum when CD = DE.

(*c*) Line FG = 1000 ft. and, if higher, will exceed 1000 ft. and, if lower, will increase borrow near F and waste near G.

(*d*) If the line HM is lowered, the borrow near M and the waste near H are decreased, but the haul is increased by trapezoidal areas of which HI, JK, and LM are their smaller bases, while it is decreased by trapezoidal areas of which IJ and KL are their larger bases. The net result is the equivalent of increasing the haul by a trapezoidal area which has an upper base of 1000 ft. and a lower base greater than 1000 ft., so that the limit of economical haul is exceeded. If the line is raised, by similar reasoning the cost of the additional borrow and waste will be greater than the saving in the haul item.

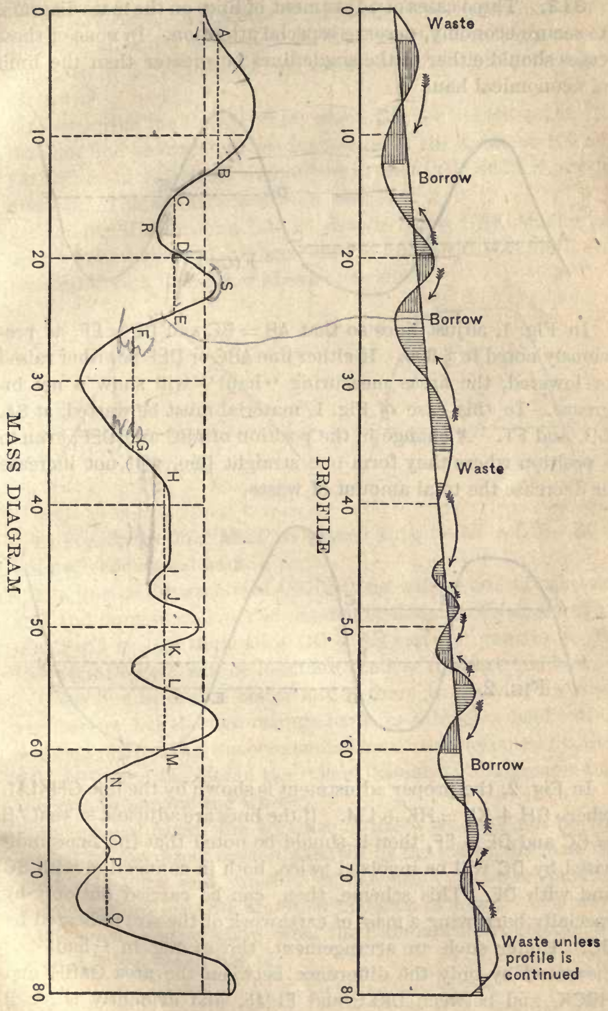
(*e*) Line NOPQ is placed so that NO + PQ − OP = 1000 ft. A change up or down will increase the cost.

(*f*) If the profile were continued beyond station 80, it is quite possible that the material indicated as waste could be utilized in fill, or part of it so utilized.

(*g*) As the profile is shown, there is a small amount of cut carried into fill close to station 80.

(*h*) The projections of the points A, B, C, D, etc., up to the profile, serve to show where material should be wasted, where borrowed ; what material should be carried forward, what backward. The study of the mass diagram has shown that the arrangement adopted is the most economical.

The exact stations of the points A, B, C, D, etc., can be determined accurately from the cross-section notes and the volumes of earthwork already computed, if this should seem desirable.


Waste
Borrow
Borrow
Waste
Borrow
Waste unless
profile is
continued
PROFILE
MASS DIAGRAM
A
B
C
R
D
S
E
F
G
H
I
J
K
L
M
N
O
P
Q
0
10
20
30
40
50
60
70
80

312. Three cases of adjustment of lines on the mass diagram, to secure economy, deserve especial attention. In none of these cases should either of the single lines be greater than the limit of economical haul.

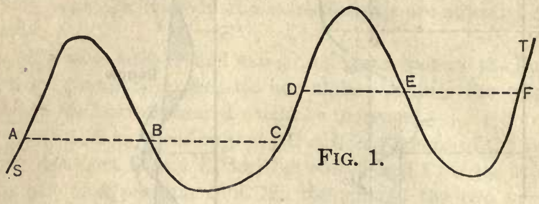


FIG. 1.

In Fig. 1, adjust lines so that AB = BC and DE = EF, as previously noted in § 306. If either line ABC or DEF be either raised or lowered, the areas measuring "haul" will show a net increase. In this case of Fig. 1, material must be wasted, at SA, CD, and FT. A change in the position of ABC and DEF, even to a position where they form one straight line, will not increase or decrease the total amount of waste.

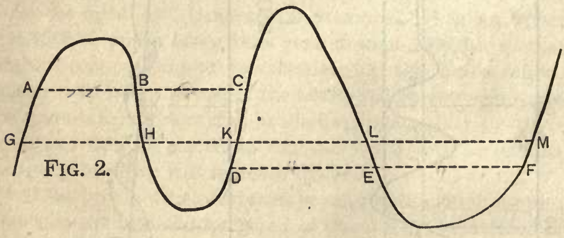


FIG. 2.

In Fig. 2, the proper adjustment is shown by the line GHKLM, where GH + KL = HK + LM. If the lines are adjusted so that AB = BC and DE = EF, then it should be noted that the mass indicated by DC will be involved twice, both in connection with BC and with DE. This scheme, then, can be carried out only by specially borrowing a mass of earthwork of the size indicated by DC. Under such an arrangement, the saving in "haul" is measured by only the difference between the area GABH and HBCK, and between DKLE and ELMF, and evidently is small compared with the cost of the extra borrow at DC.

The arrangement further is unsatisfactory inasmuch as the diagram, after all, does not show whether the actual excavation indicated by DC shall be hauled toward H, or toward L, or partly each way.

Considering the line GHKLM, where GH + KL = HK + LM, if a through line be drawn above it the sum of the areas at HK and LM will be increased more than the areas at GH and LK are decreased. The net area will be increased.

Similarly if a through line be drawn below GHKLM, the net area will be increased. Therefore the line GHKLM is more economical than a line either above or below it.

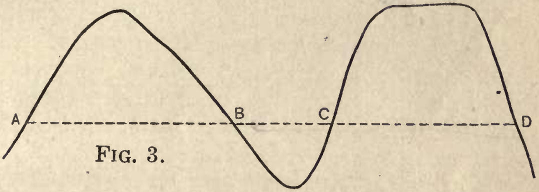


FIG. 3.

In Fig. 3, the line ABCD is placed so that AB + CD – BC = limit of economical haul.

If a line be drawn above ABCD, there will be additional waste at A and borrow at D. The net average haul on this mass above ABCD will be less than AB + CD – BC and the gain in cost in the "haul" item will be less than the loss in waste and borrow.

If the line be drawn below ABCD, there is a saving in waste and borrow but the net average haul on this mass below ABCD will be greater than the economical haul as represented by AB + CD – BC, and the loss in the "haul" item will be greater than the gain in borrow and waste.

In unusual cases, the conduct of the work may not be controlled by the direct economy shown by the use of the mass diagram. Facility and rapidity of execution or other practical considerations may dictate other methods of work. It is worth while even then to know what the economical arrangement is, even if it is not adhered to.

220

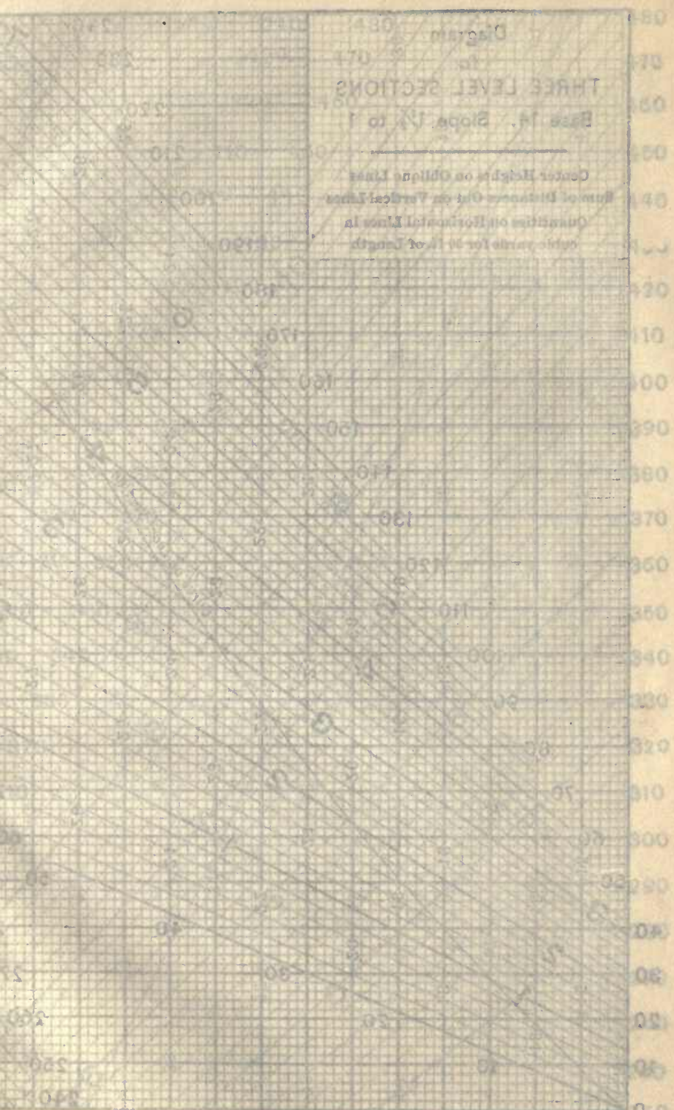


Diagram
for
THREE LEVEL SECTIONS
Base 14. Slope 1½ to 1
Center Heights on Oblique Lines
Sum of Distances Out on Vertical Lines
Quantities on Horizontal Lines in
cubic yards for 50 ft. of Length

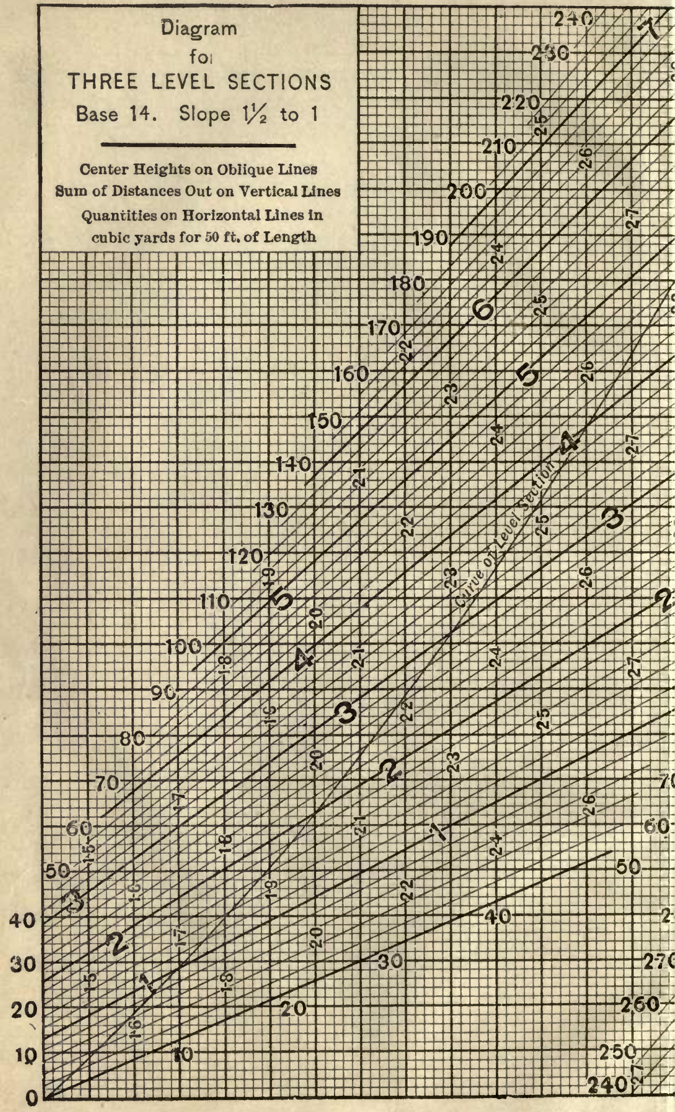
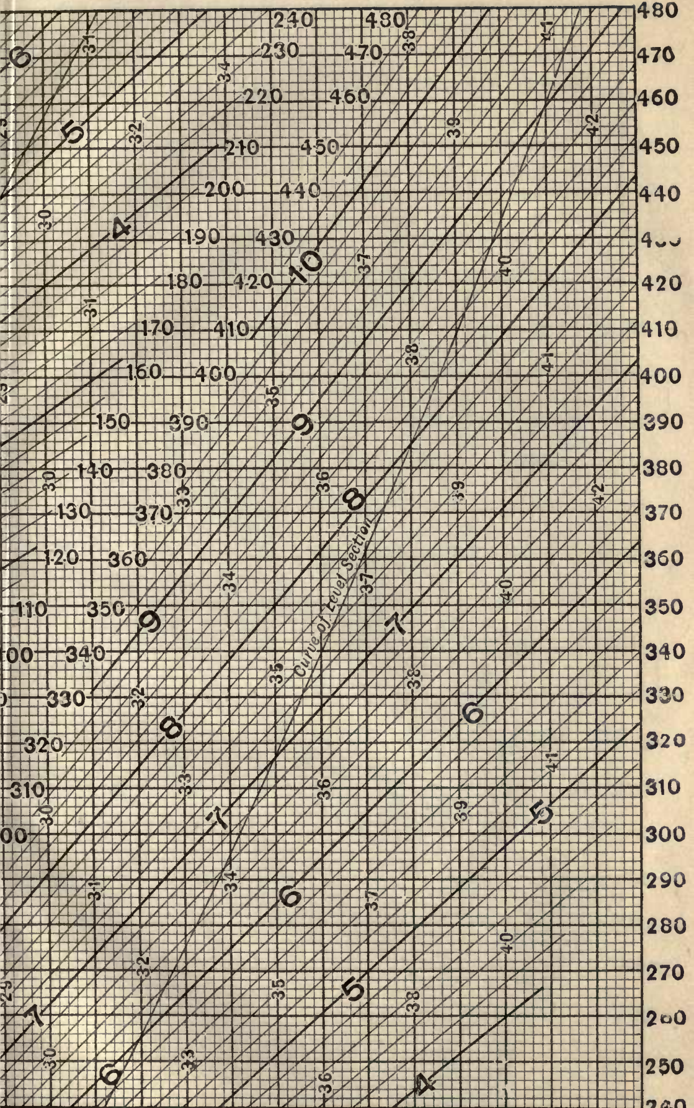


Diagram
for
THREE LEVEL SECTIONS
Base 14. Slope 1½ to 1
Center Heights on Oblique Lines
Sum of Distances Out on Vertical Lines
Quantities on Horizontal Lines in
cubic yards for 50 ft. of Length
Curve of Level Section


Curve of Level Section
480
470
460
450
440
420
410
400
390
380
370
360
350
340
330
320
310
300
290
280
270
250
240

2 6

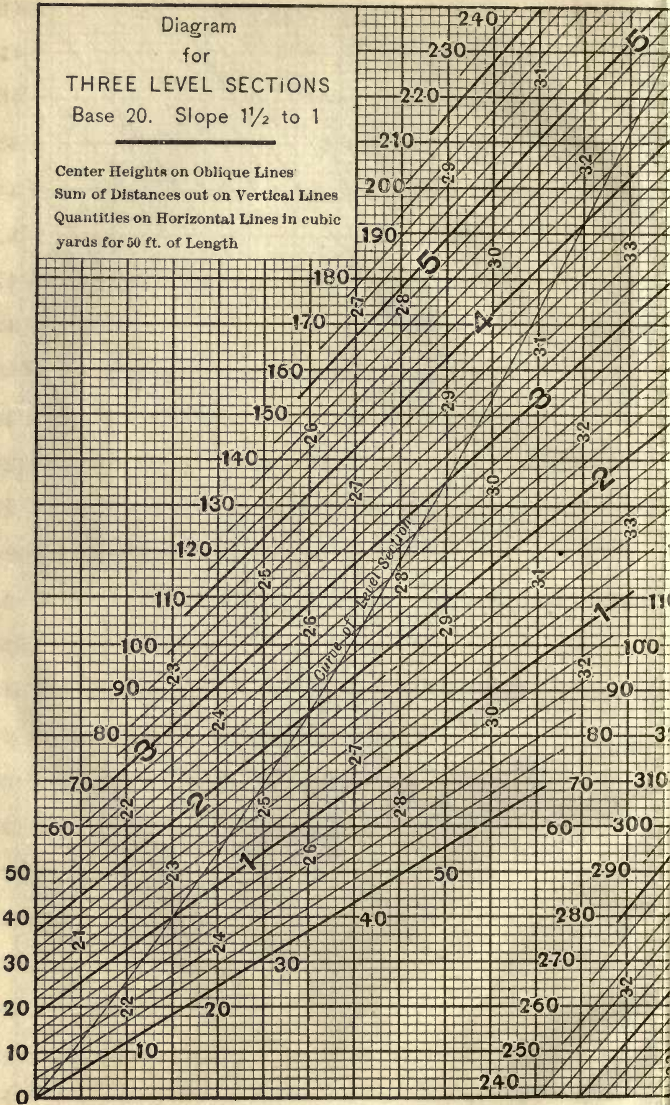
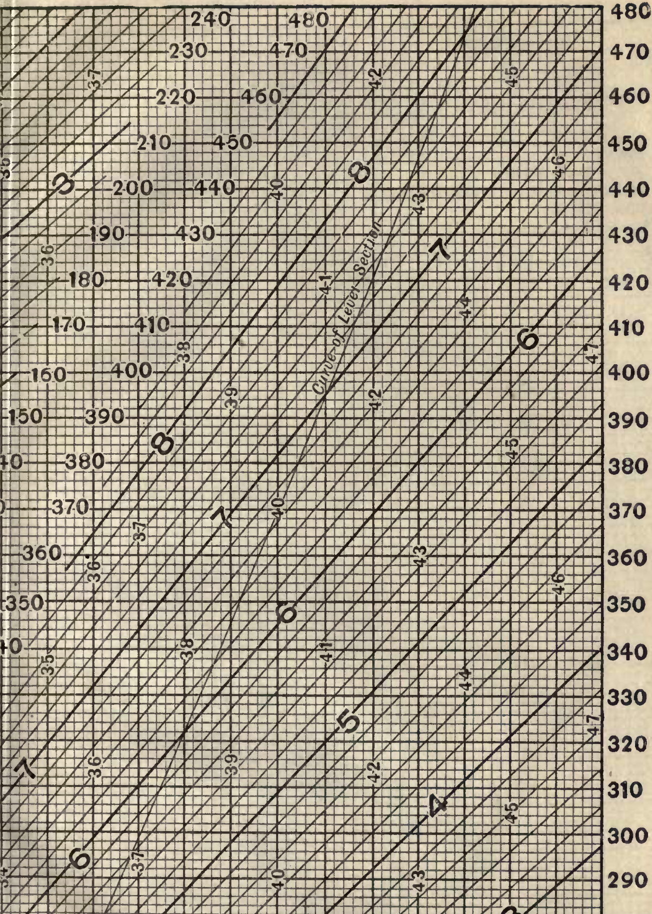


Diagram
for
THREE LEVEL SECTIONS
Base 20. Slope 1½ to 1
Center Heights on Oblique Lines
Sum of Distances out on Vertical Lines
Quantities on Horizontal Lines in cubic yards for 50 ft. of Length
Curve of Level Section


Curve of Lever Section
480
470
460
450
440
430
420
410
400
390
380
370
360
350
340
330
320
310
300
290
280
270
260
250
240

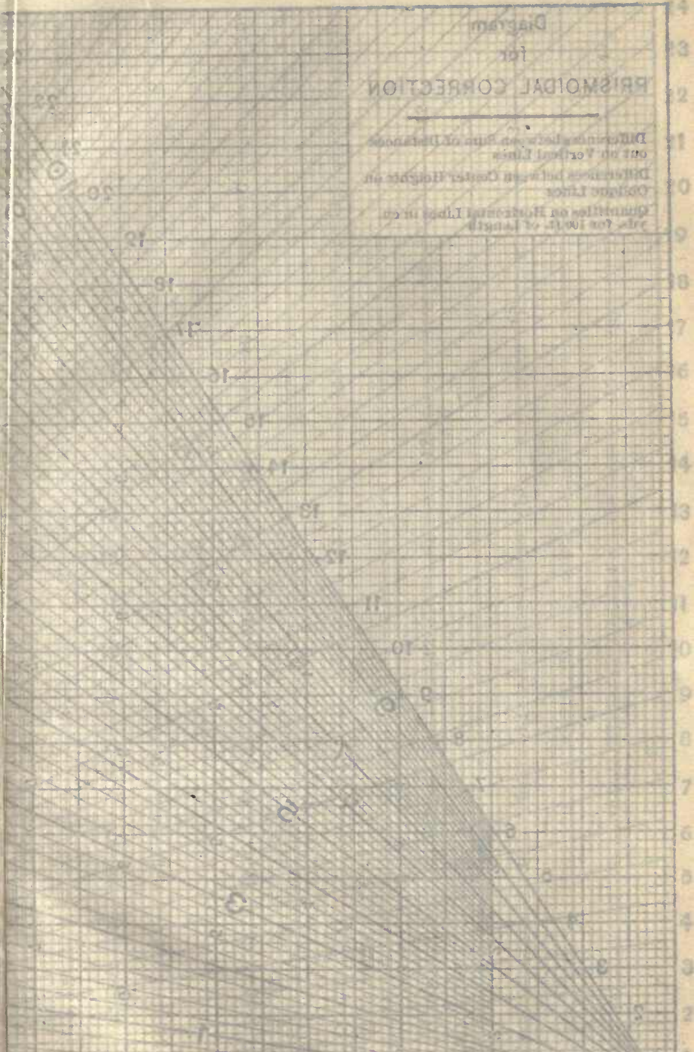


Diagram
for
PRISMOIDAL CORRECTION
Differences between Sum of Distances out on Vertical Lines
Differences between Center Heights on Oblique Lines
Quantities on Horizontal Lines in cu. yds. for 100 ft. of Length

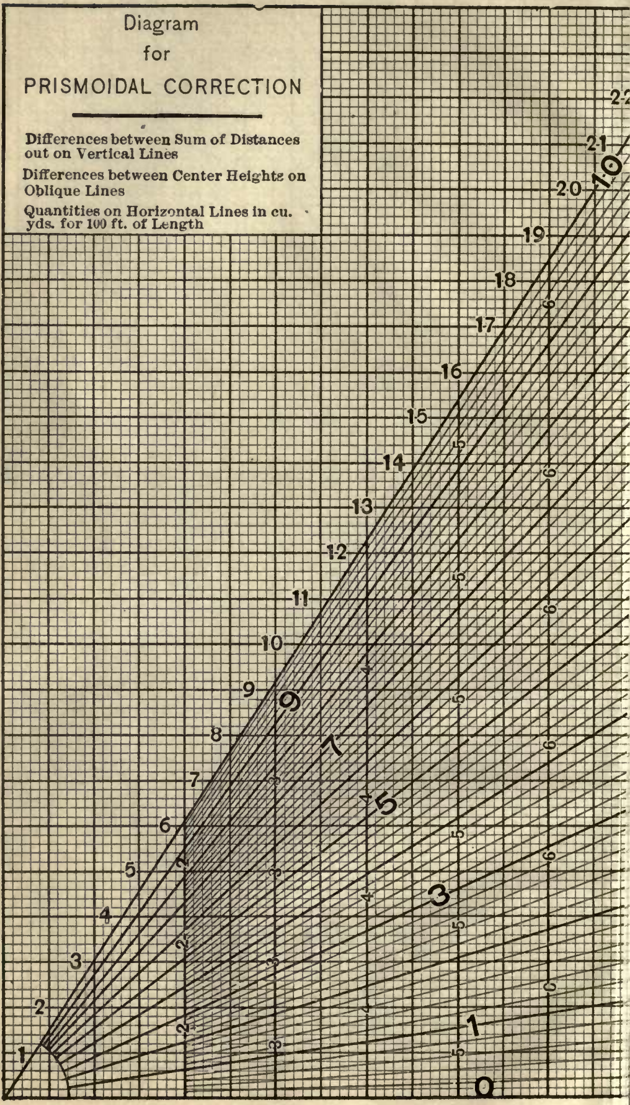
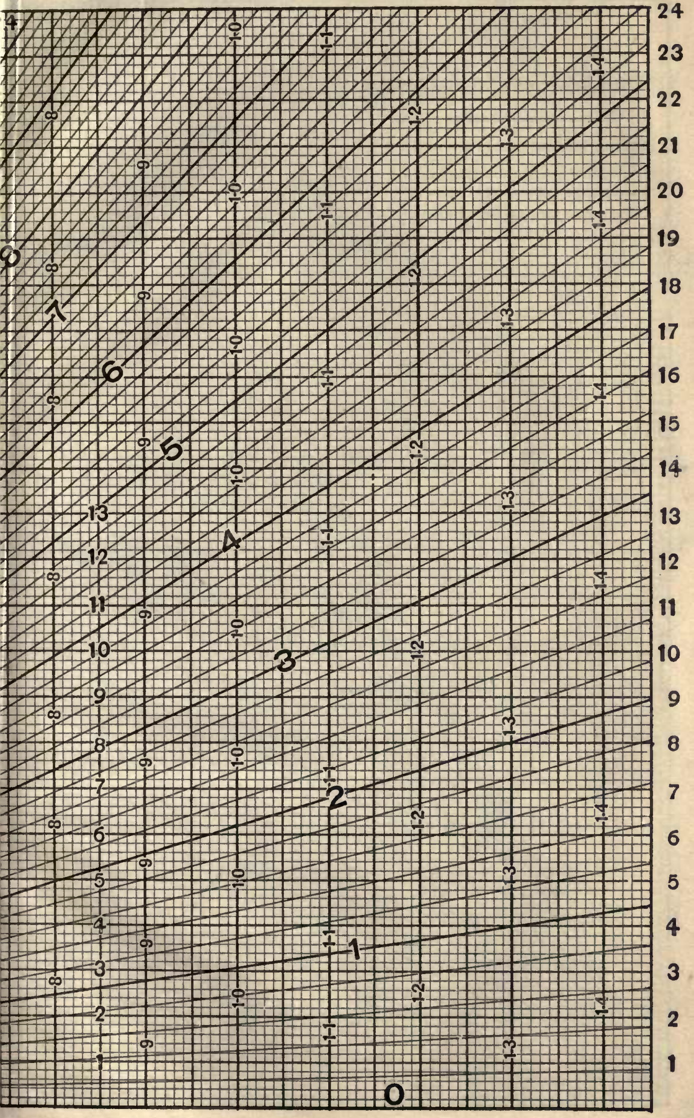


Diagram
for
PRISMOIDAL CORRECTION
Differences between Sum of Distances out on Vertical Lines
Differences between Center Heights on Oblique Lines
Quantities on Horizontal Lines in cu. yds. for 100 ft. of Length



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0
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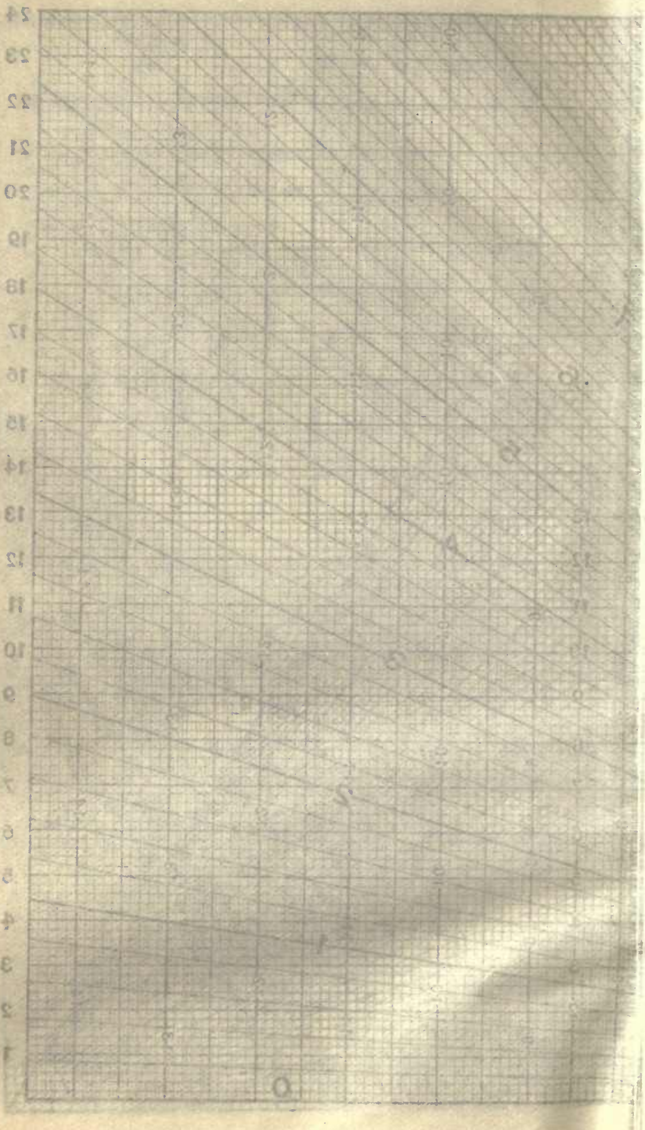
8
9
1·0
1·1
1·2
1·3
1·4



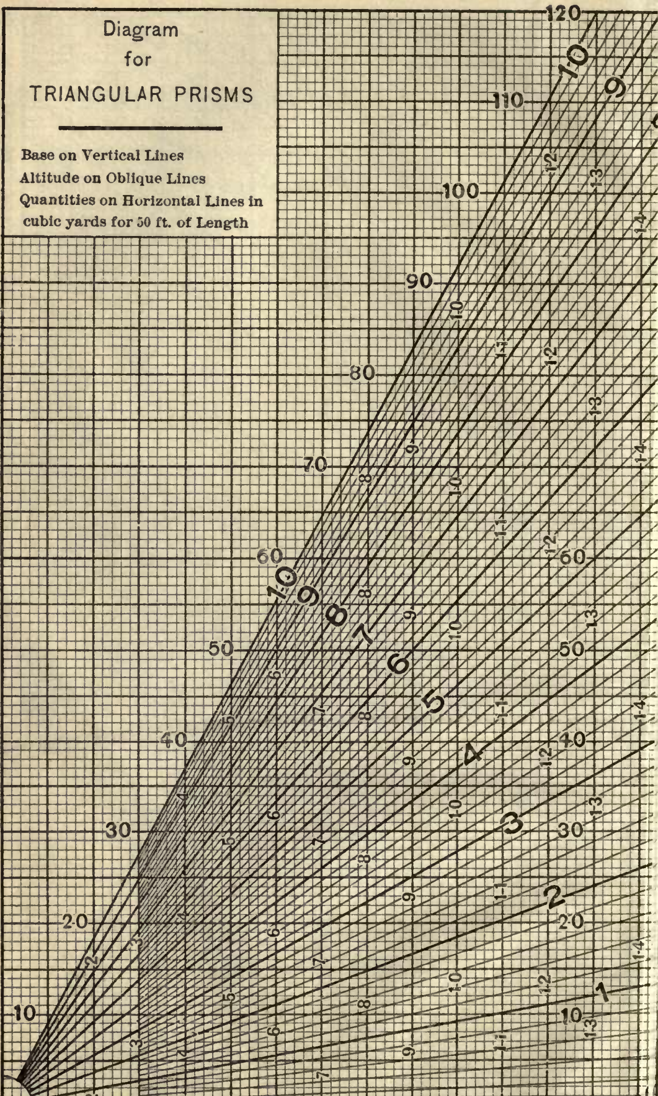
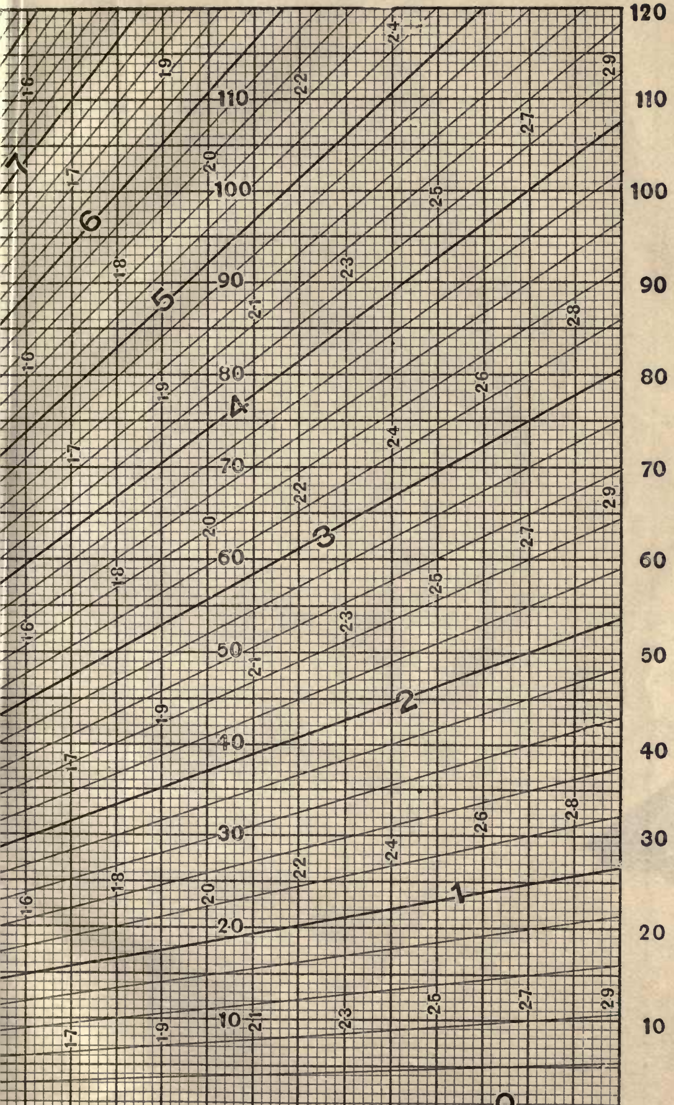


Diagram
for
TRIANGULAR PRISMS
Base on Vertical Lines
Altitude on Oblique Lines
Quantities on Horizontal Lines in
cubic yards for 50 ft. of Length



120
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100
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80
70
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50
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1·9
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